THE NATIONAL UNIVERSITY OF LESOTHO

DEPARTMENT OF PHILOSOPHY

LEIBNIZ'S *CHARACTERISTICA UNIVERSALIS* AND *CALCULUS RATIOCINATOR*: A CRITICAL APPRAISAL

BY

NTHABELENG MOLETSANE (201501166)

DISSERTATION SUBMITTED TO THE DEPARTMENT OF PHILOSOPHY IN PARTIAL FULFILMENT OF THE REQUIREMENTS OF THE MASTER OF ARTS IN PHILOSOPHY

NAME OF SUPERVISOR: DR C T ZIMUNYA

DATE: JULY 2023

DECLARATION

Student

I hereby declare that this dissertation is a product of my investigation and effort except where acknowledgements are made. This is, therefore, my original work and was not copied from elsewhere.

Signature --- Date……/……/…….

Supervisor

I confirm that this dissertation was carried out by this student and submitted for review with my approval

Signature -------------------------------------   Date……/……/…….

Abstract

This study investigates the philosophical, linguistic and computational dimensions of Leibniz's ambitious scheme of mathematizing natural language and automating reasoning for the purposes of developing what he referred to as the 'Encyclopaedia of human knowledge.' Leibniz envisioned a universal language capable of perfectly mirroring reality and representing human knowledge through symbolism; and he referred to such a language as the *characteristica universalis.* In a complementary manner, he sought to develop an automated framework for logical inferences through what he referred to as the *calculus ratiocinator.* This study appraises the feasibility of Leibniz's ideas in natural discourse. It provides an in-depth analysis of the historical perspectives about natural language from which Leibniz ideas emerged. It also demonstrates attempts made towards developing his universal language and the reasoning calculus. The study discusses the extent to which Leibniz's project has been successfully implemented in the advancement of artificial intelligence research and automated reasoning in computer science and mathematics, respectively. It also evaluates the capability of Leibniz's formalism in eliminating ambiguity, or in more general terms, semantic indeterminacy. In addition to that, the study explores certain linguistic models of ambiguity resolution which enable natural language users to use language efficiently. And in light of which, the study concludes by indicating the limitations of Leibniz's *characteristica universalis* and *calculus ratiocinator* in natural discourse. The study also highlights the significance of natural language, despite its complexities, in natural discourse settings.

Key Terms: *Characteristica universalis, Calculus ratiocinator,* Natural Language, Ambiguity

Dedication

This thesis is dedicated to my family for their encouragement and unwavering support throughout my academic journey. Thank you for your patience and for believing in me. I am forever grateful.

Acknowledgements

The accomplishment of this dissertation was made possible by the unwavering support and assistance of the following individuals, to whom I would like to express my heartfelt gratitude and appreciation:

1. To my supervisor, Dr. Zimunya, for the pivotal input he made from the early stages of this research and the invaluable discussions which helped establish new ideas for this study. This also serves to acknowledge and show appreciation for the time Dr. Zimunya has invested in my study throughout the process of conducting this research, the valuable comments, the constructive feedback, and motivation, which have contributed immensely towards the completion of this dissertation.
2. To all members of the Department of Philosophy at the National University of Lesotho who have imparted their knowledge with me throughout my years of study at the National University of Lesotho.
3. To my family, but most importantly, to my mother, 'Malerato Moletsane, and my sister, Lerato Moletsane, for their continued trust and encouragement in my academic pursuits, their moral support and their invaluable assistance towards the successful fulfilment of this dissertation.
4. To all individuals who have contributed towards the successful completion of this dissertation, and yet have not been distinctly recognized by name, thank you.

Contents

Chapter One: Overview of the Study

1.1 Background

Leibniz, in his formulation of a mathematical system of logic, idealized an algebra of thought through which all logical undertakings would be carried out by means of arithmetic calculations. He argued that ambiguities and certain other defects of natural language render it inadequate for the purposes of logic, and as such, a creation of a perfectly designed artificial language proved necessary. Leibniz was convinced that such a language would erase natural language ambiguities whilst exhibiting the logical form of sentences through the use of algebraic symbolism, and subsequently, mirror the world in a clearer and distinct manner (Lacona, 2018: 9).

Leibniz believed that all human thoughts can be reduced to primitive concepts, and in turn be designated arithmetic symbols which would be chosen arbitrarily, although, in a manner that each symbol or character signifies not more than one primitive concept. To avoid ambiguities, each complex idea would then be assigned its unique sign; and the collective set of such symbols would form an 'alphabet of human thought' (Leibniz, 1956: 342). In Leibniz's view, at their core, all human languages share an element of thought (Kopytowska and Chilton, 2018: 27). Leibniz termed the language thus resulting from the adaptation of algebraic notation as the *characteristica universalis* (or sometimes known as the *lingua characteristica* or the universal characteristic*).*

In a complementary fashion, Hintikka (1997: ix) notes that in connection to the *lingua characteristica,* the *calculus ratiocinator,* as a general system of notation, reduces the process of reasoning to computation so as to deduce the truth or falsity of symbolic propositions. Following this line of thought, Leibniz was certain that all misunderstandings would be eliminated and that defects would only be in calculations rather than in the language. The *lingua characteristica*, in Leibniz view, presupposes a universal language employing mathematical symbols in place of natural language sentences; and *calculus ratiocinator* serves as an instrument of arithmetic notation.

However, there is more to language than its logical underpinnings; and Leibniz, in his aspiration to eliminate ambiguities and imprecisions in natural language, seems to have, to a great extent, taken for granted the complexity of natural language. Signs and symbols do not capture all that there is to natural language, and subsequently, mathematizing natural language by substituting simple ideas with arithmetic symbols resembles more of a calculus for computations than a language suitable for human communication. Hence Leibniz *lingua characteristica* cannot be proclaimed as a universally logical ideal language.

1.2 Statement of the Problem

Leibniz's *characteristica universalis* has notably played an important role in the advancement of computer science. Nonetheless, an attempt to eliminate certain defects in natural language by use of arithmetic symbolism devoid of semantic content can result in an oversimplified perspective of human language. It is rather difficult to accurately capture the semantics of natural language wholly through algebraic signs. Natural language is much more complex and, not only are there ambiguities to resolve, but also other complex aspects of language that ought to be taken into consideration. Leibniz's universal characteristic and the *calculus ratiocinator* can neither represent natural language processing in its entirety through manipulation of symbols nor can they serve as an appropriate medium for expressing thoughts in natural discourse settings.

Ambiguities, either lexical or structural, are omnipresent in natural languages, and as a result, ought to be acknowledged as an inescapable component of language. Although inevitable, ambiguities can be resolved through an appreciation of specified linguistic mechanisms implemented for the purposes of filtering out unbefitting semantic and syntactic interpretations in order to comprehend sentential expressions. In general, flaws in natural language such as ambiguities, vagueness and obscurities, cannot be dealt away with by fabricating an artificial language, but rather, through an acknowledgement of the complexity of language and attainment of knowledge about the world sufficient for the purpose of understanding language as it is used by its speakers.

1.3 Aim of the Study

The aim of this study is to re-evaluate the plausibility of Leibniz's ideation of reducing natural language to algebraic formulations as a mechanism to eliminate ambiguities embedded within natural language.

1.4 Research Objectives

The objectives of this study are to:

1. Outline Leibniz's *characteristica universalis and calculus ratiocinator.*

2. Assess whether algebraic formulations in Leibniz's logic are adequate for capturing certain aspects of natural language.
3. Discuss the peculiarities in language that may lead to misinterpretation of information.
4. Discuss linguistic mechanisms employed in resolving ambiguities in natural language.

1.5 Research Questions

This study seeks to address the following research questions:

1. Can Leibniz's *characteristica universalis* accurately express sentential meaning?
2. Do the arithmetic formularizations capture inclusively all aspects of language?
3. What are some of the unique characteristics of language that may create misunderstandings if not properly decoded?
4. Which linguistic procedural mechanisms can resolve ambiguities?

1.6 Significance of the Study

In light of Leibniz's *characteristica universalis* and *calculus ratiocinator,* two schools of thought have surfaced: logic as language and logic as calculus. For the most part, logicians have predominantly concerned themselves with the contrast between these two assumptions and more often than not, have continued to reason in a way that advocates for one of the two postulations whilst, on the other hand, invalidating the other. But despite any such efforts, what is ultimately at stake is not essentially the evaluation of the soundness of either one of the two assumptions, but rather, the injustice which systematization of language, by means of algebraic symbolization, does to natural language.

Language is not merely an articulation of straightforward meaningful sentences whose truth values can be instantly determined. With this in mind, the concern that naturally arises is whether Leibniz's mathematical system of logic encapsulates the rhetoric and literal devices apparent in natural language. It is essential, therefore, to assess the capability of the Leibnizian *characteristica universalis* in an attempt to adequately express all aspects of language whilst also perfecting it.

1.7 Theoretical Framework

The theoretical underpinnings of this study this study are the semantic theory and the contextual theory of meaning, which both analyse the ability of interlocutors to understand and make inferences from sentences uttered in a language.

1.7.1 Semantic Theory

Semantics, as a sub-field in linguistics—the study of language— concerns itself with the meaning of lexical units in a given language and how these consolidate to form meaningful sentences. A semantic theory then, as defined by Katz and Fodor (1963: 176), accounts for the interpretative ability inherent in language users which aid them in identifying the content of a sentence, possible readings, and the semantic relations of lexical units in a sentence. In an actual discourse, it is not always the case that interlocutors utter straightforward sentences whose meaning can be immediately apprehended.

Often at times, natural language speakers use language in a rather complex manner, and as such, to understand uttered sentences, it would require decoding the meaning of such sentences by exercising interpretational abilities. Chierchia and McConnell-Ginet (1990: 162) note that language goes beyond simply transmitting information, thus, incorporates also the use of rhetoric devices which give an aesthetic pleasure and amusement to speakers of a language, and yet, demand a multiple level decoding of an utterance in order to understand what is being expressed. Interpreting complex sentences involves identifying and exploiting semantic relations of words in an utterance so as to eliminate any irrelevant semantic readings (Katz and Fodor 1963: 175).

The semantic theory places emphasis on the way in which speakers of language encode and decode the meaning of their sentences. A speaker of a language may deliberately construct a lexically or structurally ambiguous sentence, and the hearer of the sentence is expected to interpret the ongoing discourse by first marking the ambiguity and resolving it. The concept of semantic markers is the crux of the semantic theory.

The idea is that any vague or ambiguous lexical unit can be associated with semantic markers which specify the best possible sense of a word. The semantic markers highlight the most appropriate semantic reading in any given discourse and as such, lexical units which do not correspond with the semantic markers in an analysis can be discarded (Katz and Fodor 1963: 209). However, in cases where there are no semantic markers, to comprehend implicit sentences, there is need for contextual specification.

1.7.2. Contextual Theory of Meaning

The theory of context underpins pragmatics, which has been defined by Levinson (1983: 21) as "... the study of the relation between language and context" necessary for understanding sentence meaning. Tsohatzidis (1994: 2) submits that what natural speakers mean in asserting a sentence does not always correspond with the literal meaning of an uttered statement, and thus, it is important for one to be aware of the contextual setting in which an utterance is made in order to deduce what the speaker means in uttering it. For instance, the utterance "your hair is pretty" could be used as a form of praise at face value, but when put in context, it could be use of sarcasm by an educator to a student as an implicit order for them to trim their hair as per the school guidelines.

The contextual theory of meaning emphasizes the importance of contextual specification for the interpretation and comprehension of sentences in any given discourse. Situating sentences in the context in which they were uttered, rather than isolating them, makes it easier to understand the meaning expressed by the sentence. Chierchia and McConnell-Ginet (1990: 159) note that linguistic knowledge, more often than not, is not sufficient in understanding sentences, and as such, context plays a vital role in disambiguating sentences or adding bits of information, initially left out by the speaker, so as to unveil the meaning of a sentence.

Shuy (2001: 446) adds that to arrive at the propositional content conveyed by a sentence, a contextual parameter is essential for the purposes of understanding sentential meaning and determining the truth or falsity of such a sentence. Context does not only mean the setting, but it also denotes the information shared, the participants sharing the information, the background knowledge they share, non-verbal aspects of language involved and the intention of the discourse (van Dijk, 2008: 7-9).

In essence, the semantic theory highlights the importance of meaning in a language and speakers' ability to comprehend the syntax of such a language. Leibniz's universal calculus would be presumed merely as manipulation of symbols devoid of meaning and as such befitting for automaton and not for practical human communicative processes. De-semantification language results in an abstract calculus which is not inclusive of all aspects of language and thus, undermining one of the most essential roles of language: to convey meaning.

The contextual theory of meaning elucidates how the ambiguities in sentences of natural language can be resolved by taking into account the pragmatic concept of context. This theory places emphasis on the importance of context in disambiguating sentences. Leibniz's context-free language manipulates language in isolation, and this abstraction does not, in practical terms, eliminate ambiguities. The semantic theory and the contextual theory of meaning serve as the foundational basis of this study so as to demonstrate that Leibniz's *characteristica universalis* cannot, in praxis, eliminate ambiguities in natural language, but rather, how the abstraction of language in Leibniz logic strips language off some of its essential aspects.

1.8. Research Methodology

Of the two main research methods—qualitative and quantitative— this study adopts a qualitative research methodology. Within the confines of a qualitative framework, this study is a library-based research and therefore, textual data such as journal articles and books which promise an appreciation of the subject matter under discussion shall be consulted. In order to address the aforementioned research objectives and research questions, an interpretative paradigm and content analysis techniques shall be incorporated within the methodological approach of this research.

1.9. Literature Review

Leibniz (1666) in his book *De Arte Combinatoria* indicates that errors in reasoning are largely due to linguistic ambiguities and thus sketched his project of developing a formalized language so as to remedy the imperfections of natural language. Leibniz then sought to create a universal language whose corpus was mainly characters resembling algebraic symbols and attributed to certain definite concepts. This artificial language, as Leibniz conceived, would mirror reality more distinctly and also explicate the logical form of sentences through arithmetic symbolism. Leibniz labelled his symbolic language as a *characteristica universalis.* The vocabulary of such a formal language would be signs and symbols whose function would be similar to those numerals in mathematics such that whenever there are disputes; calculations would be made so as to settle any ongoing controversies. For such calculations to be made, Leibniz introduces a *calculus ratiocinator* which, in connection to the universal characteristic, serves as a calculus that detects and cultivates errors in reasoning and consequently, determining the truth or falsity of logical propositions.

Boole (1847) in his book *Mathematical Analysis of Logic,* presents an attempt to develop a reasoning calculus which was an advancement of Leibniz's *calculus ratiocinator.* He argued that arithmetic symbols and mathematical principles express and specify the logical structure of propositions and the relationship between such symbols. Like Leibniz, Boole's algebraic approach to logic brings into perspective how simple propositions of natural language can be arithmetized through binary numeration. His calculus of logic employed binary arithmetic (1's and 0's) such that, for the purposes of clarity, the meaning of propositions should match the designated meaning of 1 or 0. Boole symbolized variables representing concepts as 'x,y,z,' and then introduced operators that highlight the relations between symbols (+ , -, =). He indicated that the logical soundness of an argument can be assessed if and only if such an argument can be symbolized algebraically in order for it to be mathematically computed so as to determine whether it is sound or unsound. In general, Boole argued that the essence of a logical reasoning calculus is mainly embedded within the algebraic structure of propositions and their axiomatization, which make possible the attainment of logical truths.

In his *Begriffsschrift,* Frege (1879 [1997]), rejects Boole's conceptualization Leibniz's *characteristica universalis* and *calculus ratiocinator* and his efforts towards its actualization. He argues that the Boolean algebraic approach results in a tool for undertaking meagre computations — *calculus ratiocinator*— but neglects foundation of Leibniz's mathematical system of notation which is the universal characteristic; and as such, he concludes that Boole's reasoning calculus cannot, in itself, be considered as a perfect and adequate representation of Leibniz's project. In light of this, Frege presents his own understanding and what he thought to be a perfect realization of the Leibnizian mathematical system of logic. With this in mind, Frege argued that his concept-script is two-dimensional since, unlike the Boolean algebra, it is not only limited to the development of a *calculus ratiocinator*, but it also constructs a *lingua characteristica*.

For Frege, it is erroneous to de-semanticize natural language by merely substituting primitive concepts with symbols devoid of meaning as Boole did, but rather, he argues that fabricating a universal language and a reasoning calculus demands that the arithmetic symbols should be capable of explicitly expressing thoughts so as to eliminate indefiniteness in meaning. The contrasting element between Boole and Frege's mathematical language is that the latter's variables have semantic content whilst in the former's symbolism, variables possess no semantic content of their own because their meaning differs over various domains of applicability. Frege's concept-scripts presents the form '⊦Γ' which indicates that a logical proposition has judgeable content such that it can be classified as either being true or false. In connection to this, Frege was convinced that the contentful symbolic variables presented a perfect representation of Leibniz's *characeristica universalis* whilst the logical operatives and laws governing them presented a *calculus ratiocinator*. Although both Boole and Frege aimed at eliminating inconsistencies of natural language, Frege, following his rejection of Boole's ideas, confidently proclaimed his *Begriffsschrift* as a perfect approximation of Leibniz's universal language.

Sourcing his inspiration from Frege, Wittgenstein (1921[1999]), in his *Tractatus Logico-Philosophicus,* acknowledges that logic is both a truth-purifying tool and a channel through which ambiguities can be eliminated from natural language; and in a similar manner as Frege, he sought to further extend logic far beyond Boole's algebraic approach. However, Wittgenstein emphasized the view that one cannot escape natural language since their understanding of language and logic is confined within the bounds of their natural or ordinary language. Following this line of thought, he challenges the universalist conception of language by arguing that for one to analyse ordinary language, re-interpret its semantics and fabricate a universal language, one would have to step outside ordinary language, and for him, this is not feasible. In a similar as the aforementioned philosophers, Wittgenstein was aware of the inconsistencies and obscurities embedded within ordinary language but to resolve these flaws, he proposed the notion of 'language games' which explains the way in which natural language users use language in ordinary discourse, and as such, it would serve a tool through which humans can come to appreciate the linguistic components of natural language and the logic embedded within it.

Lichtenberg (1800-06 [1984]) in his book *Sudelbücher* informs his readers that he is aware of the imperfections embedded within natural language. However, he strongly rejects any attempts to substitute natural language with a logically perfect language. He argues that it is not necessary to fabricate an artificial language aimed at cultivating errors in reasoning since ambiguities and all other defects in natural language are an important part of language. Lichtenberg argues that language, despite its imperfections, plays an essential role in making possible the attainment and communication of knowledge. In light of this, he denounces Leibniz's idea of creating a formalized language on the ground that it fails to appreciate the transcendental functions of ordinary language.

In his renowned paper, *Logic as Calculus and Logic as Language,* Heijenoort (1967) accounts for the development of Logic following the Leibnizian idealization of a mathematical system of logic incorporating both a universal language and a reasoning calculus. The difference in comprehension of the Leibnizian notions —*characterica universalis* and *calculus ratiocinator*— apparent between Frege, Boole and other collaborators, has resulted in these notions being treated as two independent and contrasting logical systems. Heijenoort presents Boole as the proponent of the view that Logic is a calculus whilst Frege as the advocate of the idea of logic as a language. In connection to this, he presents a comparative analysis of the dichotomy between the two assumptions by noting that the Fregean approach places emphasis on the logico-grammatical elements of a universal language, whilst on the other hand, the Boolean approach is more of an abstract calculus. Heijenoort concluded that the two assumptions have been strongly argued for or against since the advent of Leibniz and thereby resulting in the formation of the two traditions of logic.

Hintikka (1997) in his book, *Lingua universalis vs. calculus ratiocinator: An Ultimate Presupposition of Twentieth-Century Philosophy,* adopts Heijenoort's idea of the two approaches to logic and presents them as two ways in which language can be perceived. He presents the view of language as a universal medium, and language as calculus. Hintikka notes that the former view presupposes language as an inescapable prison in the sense that one cannot step outside their language and reinterpreted its semantics. On the other hand, the latter approach assumes language to be a reinterpretable calculus whose semantics can vary over different domains of applicability. In light of this, Hintikka's main concern was to determine which of the two views of language is sound.

Against Leibniz and his collaborators, Sowa (2000) in his book, *Knowledge Representation: Logical, Philosophical, and Computational Foundations,* argues that assuming the imperfections of natural language can be remedied by merely developing an exact and perfect language is misguided and erroneous since the apparent inconsistency of language results from the complexity of reality itself rather that the linguistics of natural language. He argues that ambiguities arise when a single word is used to describe different situations in reality, however, such inconsistencies are not to be thought of as defects that can be eliminated but rather, aspects of language that can be resolved so as to facilitate effective communication of ideas.

In so much as the works reviewed in this study are not exhaustive of the literature on this subject matter, they however provide sufficient ground for identifying the gap or problem which this research seeks to address. As it has already been pointed out, the focal point of some of the prominent works has been mainly to argue for an algebraic approach or a Universalist approach to logic, which have been derived from the two notions of Leibniz's project— *characteristic universalis* and *calculus ratiocinator*— so as to perfect the imperfections of natural language which Leibniz himself regarded as an obstacle to the attainment of knowledge. This study appreciates all efforts to mathematize natural language but seeks to appraise the feasibility of a mathematical language in praxis, or more specifically, in natural discourse settings.

1.10 Scope and Limitations of the Study

This study concerns itself with articulating Leibnizian *ars characteristic* and discussing the dichotomy between *characteristica universalis* and *calculus ratiocinator*, and whether a mathematical language resulting from the two notions can serve as an adequate means of expressing thoughts in ordinary discourse settings in comparative to natural language. In light of this, this study does not intend to give a detailed account of the history of modern logic nor is it exhaustive of the topic under discussion.

1.11 Layout of Chapters

For the purposes of achieving the objectives, this study has been divided into five chapters which are arranged as follows:

Chapter one is the research proposal which provides an overview of this research. This chapter provides background information on the topic and states the problem to be addressed together with the objectives which this study seeks to achieve. In addition to that, the theoretical framework follows in the similar manner as research questions, the methodology adopted by the study, aim, scope and limitations, justification, as well as reviewed literature.

Chapter two outlines Leibniz's analysis and attitude about natural language which indicate the reasoning behind his conclusion that natural language is inadequate for the purposes of logic. In connection to this, the Leibnizian ideation of a combinatory method consisting of both a *characteristica universalis* and a *calculus ratiocinator* shall be discussed at length in this chapter.

Chapter three presents the importance of Leibniz's idea of precision in communication and mathematical notation. The discussion centres mainly on automated reasoning which allows for natural language statements to be translated into formal language in an attempt to eliminate indefiniteness, thus making it easier for automated reasoning programs to manipulate signs and symbols of Leibniz's universal language through arithmetic notation in order to arrive at a conclusion.

Chapter four demonstrates the repercussions of constructing an artificial language which reduces ordinary language to arithmetic symbols as in natural discourse. This chapter commends on the importance of literary and rhetoric devices such as satire, sarcasm, irony, hyperbole, banter, etc., in natural language which mathematical symbolization fails to acknowledge and appreciate as part of natural language. The soundness of the conviction that arithmetic symbols can accurately express sentential meaning and do away with ambiguities is also assessed. Chapter four also highlights some of the linguistic procedures available for deducing meaning from ambiguous and vague sentences in natural language. It discusses certain pragmatic concepts which aid speakers of a language in resolving ambiguities in any given discourse. This chapter further demonstrates the non-fulfilment of Leibniz's *characteristica universalis* erasing ambiguities in natural language as he had aspired.

Chapter five provides a condensed summary of the study and presents the researcher's reflection on the previous chapters. This chapter discusses of the main arguments presented in each chapter, following which, concluding remarks of the entire research shall be rendered.

Chapter Two: Towards Mechanizing Thought

2.1 Introduction

This chapter outlines Leibniz's percipience of natural language where it pertains words and concepts. It also provides an overview of Leibniz's conceptualization of a logical system, combinatory of both a *characteristica universalis* and a *calculus ratiocinator*, which lays groundwork for automated reasoning.

2.2 Leibniz's Analysis of Natural Language

Pombo (1987: 31) says that natural language is a primitive reconstruction of the Adamic language and thus defines it as the spontaneous or impulsive articulation of thoughts through linguistic expressions which do not, in a strict sense, claim universality, although common to all humans. The estimated number of natural languages spoken all over the world is over 7000, according to Ethnologue. Lichtenberg (2012: 81) asserts that human knowledge is embedded within natural language, and as such, all attempts to know follow from the use of language. The main point here is to highlight the role language plays in understanding reality. In light of this, Leibniz aimed at realizing a general science which embodied human knowledge in its entirety hence his analysis of natural language precedes the formulation of his project.

Leibniz observes that language mirrors reality and it is through it that thought is made possible; however, he laments that it is the very same language that is responsible for erroneous reasoning. Leibniz argues that often when humans find themselves engaged in controversies, it is mainly on account of the ill usage of terms or misinterpretation of judgments (NE: BK III, CH. X, §4). Concurring with Leibniz, Bacon (NO: BK I, XLIII) in his theory of idols, clarifies that absurdities that are often assigned to reason are often perpetuated by language in light of the interlocutors' unfit choice of words which then give rise to confusion which ultimately fuels empty disputes amongst men.

Natural language propositions are, more often than not, open to various interpretations thus leaving room for numerous meanings to be deduced from one given statement. For instance, the sentence "Evie hit a girl with an apple" can be interpreted to mean that Evie hit a girl using an apple or that Evie hit a girl who was holding an apple. In light of the ambiguous phrase "with an apple", it becomes difficult then to determine the meaning conveyed by the proposition, which in turn obstructs any attempt to logically determine the truth or falsity of the statement. Although it may seem trivial to ponder on the imperfections of language, for Leibniz, it was imperative to perfect language for the purposes of scientific progression.

Leibniz emphasized that vagueness, obscurities and ambiguities cripple our ability to decode sentential expressions with precision, thus deterring any attempts to logically determine the truth-value of propositions. As such, he was convinced that errors in reasoning derive from natural language. In addition to that, Lichtenberg (2012: 100) argues that falsehoods in philosophy are due to language hence he says, “we cannot reason, so to speak, without reasoning falsely.” Leibniz understood language not merely as a means for facilitating communication of thoughts, but also, as indispensable for cognition. Herder (1799: 274) concedes that language and reason are inextricably inseparable; and Trauth (1989: 411) adds on to say that without language there would be no thought and vice versa.

Although Leibniz does not explicitly ascribe to the notion of the impossibility of reason in the absence of language, he acknowledges the relationship between thought and language hence his linguistic analysis of natural language. Leibniz argued that in order to do away with erroneous reasoning, it would be necessary to devise means to eliminate natural language imperfections so as to make possible the applicability of logic on propositions. One alternative would be that in order to disambiguate words and the informational content expressed by statements, words should be assigned univocal definitions. However, this is implausible because the very words employed in defining concepts are equivalently plagued with imprecisions, thence leading to an infinite regress of definitions.

Convinced that natural language is inadequate for logical operations, and devoted to eliminating natural language deficiencies, Leibniz proposed a creation of formal language such that signs or characters would represent simple ideas, and whose combination would result in algebraic formulations. The precision of such characters would remedy the obfuscation embedded within natural language, and thereby adequately demonstrate and advance the totality of human knowledge.

Despite the fact that Lichtenberg concurs with Leibniz on the fact that natural language is flawed, he vehemently opposes Leibniz’s idea of how the inconstancies and irregularities of natural language render it illogical, and on that account, necessitating a fabrication of an artificial formal language. Lichtenberg (1800: 101) illustrates that natural language flaws are not an obstacle to the applicability of logic, but rather, facilitate effective communication and cognition. He argues that language is prior to reason, therefore, it is not entirely feasible to substitute natural language by constructing a language said to be both perfect and logical (Lichtenberg, 1800: 103). Lichtenberg maintains that a linguistic analysis would suffice in clarifying equivocal expressions in natural language instead of substituting natural language with a logically perfect language.

Nonetheless, Leibniz maintains that to rely on natural language for the purposes of logic, as Lichtenberg demands, results in distorted conclusions and often unnecessary controversies. He therefore sought to create a language devoid of erroneous misinterpretations: a language through which, if intellectual disputes were to ever arise, humans would resolve them through calculations. This language, as Leibniz idealized it, would adopt a mathematical notation and through it, the kind of certainty realized in mathematics, would be realized in philosophy too.

2.3 Preliminary Philosophical Languages

In line with the assumption that natural languages are inadequate for logical proceedings and as a result halt the advancement of the body of scientific knowledge, philosophers and mathematicians took it upon themselves to create languages so as to resolve natural language deficiencies. This project of developing artificial languages involved the codification of signs which involved assigning certain ideas representative signs and specifying rules through which such signs would be combined.

Liuxiang (2000:172) explains that the first requirement towards constructing a language devoid of natural language defects, according to Bacon, is first to identify real character which, in reality, represents intelligible objects, and not mere symbols denoting initials of every noun that there is in a language. Delgarno also notes that the characters of the said language should not only accommodate all human knowledge, but there must also be a grammar governing the combination of such characters in order to make possible explicit meaning (Eco, 1995: 229).

Descartes (1981: 6) opined that although the language may appear possible to construct, it may be useful in principle, but it would be impossible to use such a language and as such saw no prospects for it in the practical sense. However, this did not deter attempts on constructing universal languages. Dalgarno and Wilkins are some of the notable philosophers who had made attempts on creating artificial languages for the purposes of facilitating accurate communication. They started first by classifying concepts into multiple genera depending on the nature of concepts and also, stipulating rules for combination of such letters. Both Wilkins and Dalgarno aimed at developing a symbolic language which would not only be inscribed but also spoken. Leibniz remarked that the languages presented by Delgarno and Wilkins were lacking since they only focused mainly on the communicative function of language, and neglected an essential function of language which is to facilitate reasoning.

2.4 Leibniz's Ars Characteristic

Leibniz was fully aware of the above-stated efforts made in an attempt to create an exemplary universal language, and as a result, he revealed their shortcomings and then presented his own version of a formal symbolic language. The *ars characteristic* is a general term referring to his combinatory method of inquiry. It is grounded mainly in the fact that it entails both a language for communicating thoughts, and a calculus capable, not only of directing the course of reasoning but also, essential for judgment. He then proceeds to outline the scheme of a universal language and reasoning calculus.

2.5 On the Universal Characteristic

Leibniz communicates his idea of inventing an ideal language exhibiting algebraic formulations, and whose realization, would perfect the human mind by exonerating it from error, thus professing pre-eminent certainty of the scientific knowledge (Leibniz, 1989: 166). The *Characteristica universalis,* as conceived by Leibniz, transcends the mere communicative function of natural language since it would also adequately represent thought through symbolism and rigorously establishing truths by subjecting specified conjectures to computation.

Liuxiang (2000: 170) reports that Leibniz's *characteristica universalis*, in Leibniz's mind, had three functions. First, he notes that it would serve as a universal language through which all sciences worldwide find their unification given that the universal characteristic makes possible the translation of all human knowledge into learnable signs. Secondly, it was intended to function as a logical system that would lead to discoveries of new phenomena which by means of logical reasoning can ultimately be discarded or confirmed as vital to the body of knowledge. Lastly, Liuxiang notes that the universal characteristic would operate as a symbolic logical system underlying all scientific investigations by prescribing certain axioms as a model thereof.

Leibniz thought that if he were to adopt a mathematical model in creating his artificial, yet logically perfect language, when implemented over various domains of applicability, it would yield the same outcomes as mathematics. Pombo (1987: 71) identifies that although Leibniz's universal characteristic is furnished on the basis of a mathematical paradigm, it should not be misconceived as merely an extension of mathematics; but rather, it ought to be appreciated as a language underlying mathematics since the world is, in itself, organized mathematically. For this reason, Leibniz believed that his universal language would guarantee the attainment of possible truths about reality with utmost precision.

To attain this level of precision, the construction of Leibniz *characteristica universalis* would be executed independent of natural languages because they have, from the onset, been classified as illogical and imperfect (Pombo, 1987: 70). "It would be a language whose structure and components would mirror directly the structure of our ideas and ultimately reality, contrasting with the unreliable and vague language of the average man" (Kanterian, 2012: 6). Leibniz remarks that the *characteristica universalis* will not only be easy to learn but also, it would be devoid of error and accordingly facilitate reasoning. However, Lakatos (1977: 10-11) cautions that mathematics, which informs Leibniz's idea of mechanizing thought, is dynamic and self-correcting: that is to say, discoveries of conjectures and their proofs are not immutable truths as projected by Leibniz.

Leibniz held firm the idea that signs were an essential element to the thinking process; hence his construction of the universal language would involve assigning symbols or characters to concepts in order to formulate formulae expressing thought. Dascal (1987: 47) adds on to say that Leibniz had stressed the value of signs in both memory and certain other 'operations of the mind.' Signs and characters serve as the lexicon of Leibniz universal language and when attached to each other, they form the syntax of the language; and it is only then that the Leibnizian calculus can determine the truth-value of the ideas communicated and signified therein.

2.6 Signs and Blind Thought

Leibniz argues that if humans were to rely solely on words for reasoning, it would be ineffective given one would then be required to know all definitions of words and recall them constantly; hence he acclaims signs for their usefulness both in memory and in blind thought. Leibniz opined that substituting words with signs would widen the scope of applicability of logical reasoning, and subsequently, prompt the discovery and validation of knowledge. Jolley (1995: 44) adds on to say that the prospects of blind thought— manipulating signs or characters— in place of conscious cognition of ideas enhances the capabilities of the human mind.

The preference of signs over words accentuates the role of signs in reasoning. Antognazza (2016: 15) explains that most of our reasoning occurs in 'blind thought' where our thinking is a result of symbolic operations without reflecting as to whether or not, the signs correlate with concepts denoting actual beings. Leibniz's idea of blind thought and emanates from his knowledge of how, in mathematics, one is able to apprehend both simple and complex mathematical procedures. In connection to this, he was convinced that the same role played by numbers in mathematics, so can the signs play in reasoning.

Premised on the notion that human thoughts are to a great extent symbolically processed, Leibniz found it befitting not only to reason with characters, but also, to use signs instead of words when articulating thoughts. Liuxiang (2000: 173) reveals that proposals were made to adopt the Chinese language as signs to replicate thought; however, Leibniz rejected this initiative on the basis that Chinese symbolism (人, 汉, 雨, 理) signifies things not ideas or basic concepts. Following this, Leibniz's burden was to identify signs and symbols that constitute the mental alphabet.

2.7 The Alphabet of Human Thought

Leibniz conception of an alphabet of thought is premised on the idea that all complex ideas can be reduced to primitive concepts, which would then be replicated in the mind as characters which form an alphabet of cognition. Couturat (1903: 430) defines the alphabet of human thought as "the catalogue of those concepts which can be understood by themselves, and from whose combinations our other ideas arise." These conceptual primes denote human thoughts in their simplest form and thus, when assigned distinctive signs and relations, complex ideas can be formulated.

Leibniz (1989: 230) highlights the point that it is from the combination of conceptual primes that the derivative notions result, and that from these derivative concepts, more composite concepts arise. Wierzbicka (2001: 232) explains that from a linguistic perspective, the alphabet of human thought as articulated by Leibniz, presupposes that there is a universal innate mental alphabet which provides sufficient ground for the analysis of human thoughts. Leibniz explains that the primary concepts are, in themselves, indefinable and thus mark the limit of human understanding, for if they were, nothing could ever be comprehended.

In formulating the alphabet of thought, as it were, Leibniz begins by classifying conceptual primes into a genus so as to bring to light the variation of each genera. He points out that characters such as *a, b, c, d, e, f,* etc. occupy the highest genera in the catalogue of concepts; and they are in their nature, as numbers, infinite (Leibniz, 1989: 229). The second genera Leibniz refers to as 'binions' for it is derived from the combination of characters in the first genera: *ab, ac, bd, cf.* The third genera consist of characters such as *abc, bdc, cde*, etc. and he refers to this genera as 'ternions.' Just as in mathematics, Leibniz (1989: 229) notes that "the prime numbers can be taken as the highest genera, since all even numbers can be called binaries, all divisible by three, ternaries, and so forth."

In light of this, human thoughts can be transcribed symbolically and ultimately exhibit the *characteristica universalis*. To exemplify this, given a proposition "All men are mortal," the initial assumption is to substitute the concepts 'men' and 'mortal' with '*a*' and '*b*' respectively. To explicate the relations and deduce truths of symbolic propositions, Leibniz introduces the *calculus ratiocinator.*

2.8 The Calculus of Thought

As Leibniz had envisioned, thoughts would assume a mathematical form; and in order to establish the soundness of thoughts then, a *calculus ratiocinator*, taken to resemble all mental operations, would display arguments in the form of calculations, as substantial proof or refutation of the expressed thoughts. Anellis (2015: 142) notes that Leibniz's *lingua characteristica* and the *calculus ratiocinator* are two aspects of a logical system and their unification results in what is called '*mathesis universalis*.' This stresses the interdependence of Leibniz's calculus and the underlying universal characteristic.

In exemplifying his calculus, Leibniz adopts Aristotle's categorical logic, and he notes that given any universal affirmative proposition 'all humans are animal,' when translated into symbolism, it will be expressed as '*a* is *b.'* Leibniz's logical calculus employs ∞ and $\oplus$ as operators, where ∞ implies that 'terms' are the same and as such, they can both be substituted in place of the other without negatively affecting the truth-value. For instance, as Leibniz (1989: 371) writes, $A \infty B$ means A and B are the same hence "[i]f $A \infty B$, then also $B \infty A$." On the other hand, $A \oplus B$ denotes that A is contained in B. Truemper (2010: 5) points out that in modern logic, $\oplus$ denotes both 'or' and 'and.'

Leibniz (1989: 240) notes that 'all' is a universal sign and is to be understood as always prefixed hence needless to symbolize it. So, for example, if 'all mammals are animals', and 'all humans are mammals', 'therefore all humans are animals.' According to Leibniz, this argument is symbolically transcribed as "if *a* is *b*, and *b* is *c*, then *a* is *c*" (Leibniz, 1989: 241). Antognazza (2018: 187) asserts that for Leibniz, the quantifier (all) is dropped is symbolism and that the transcription '*a* is *b*' does not necessarily imply that '*a* and *b*' are undifferentiated; but rather, highlights the binary relation between the basic elements of thought. Leibniz concerned himself only with universal affirmative statements and made no attempt in determining the logical form of the other three categorical statements: universal negative statements, particular affirmative statements, and particular negative statements.

In his mathematical system of logic, Leibniz (1989: 235) refers to the primitive concepts as 'terms' and notes that a categorical proposition, such as 'all men are rational animals,' has two terms — subject and predicate— and each shall be assigned a letter for symbolism and a characteristic number for computation. For instance, the term 'rational' is to be designated by a sign '*a*' and a number 2, while animal, in the same manner, a sign '*b*' and a number 3. The binary relation '*ab*' or $2 \times 3 = 6$, which is the value of '*h*' (Leibniz, 1989: 235). Therefore, the characteristic number of man is 6. Leibniz alludes that by arithmetizing logic, what would have required men's effort can be tersely shown as a whole effortlessly (Leibniz, 1989: 235).

Strickland and Lewis (2022: 23) point out that Leibniz developed a binary arithmetic which served as the basic algorithm for designing "machines that could, in principle, carry out binary arithmetic calculations with minimal human intervention." Leibniz himself would later invent a machine and Truemper (2010: 5) recounts that Leibniz calculating machine was capable of executing mathematical procedures of addition, multiplication, subtraction and division, which in turn assured him of the possibility of a computational process that could encode human knowledge and reduce all rational deliberations to calculations.

Leibniz (1989: 368) explains binary arithmetic as an isomorphic representation of natural numbers by means of 1's and 0's, where 1 denotes 'God' and 0 denotes 'nothingness.' Binary code then is a language of numbers capable of encoding various potential interpretations with precision. Leibniz observed that all other numbers are made up of the numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9; and from this, all other numbers can be formulated. However, he asserts that in binary progression, there is no need for other numbers expect 1 and 0. For Leibniz, in binary numeration, a decimal sequence of numbers 1, 5, 9, 17, 25, 37, 49 is represented as:

Decimal number	1	5	9	17	25	37	49
Binary number	1	101	1001	10001	11001	100101	110001

If characters are not used as specified, in their place, numbers will be assigned for the purposes of calculation. In the same manner as the universal characteristic, the binary system is restrictive and succinct. For instance, given a particular affirmative proposition 'Some men believe in afterlife,' the meaning of terms to be assigned numbers must either align precisely with the digits, 0 and 1, and whatever that does not fit shall be discarded. 'Men' and 'afterlife', since they are terms, shall be assigned the number 1 while the copula 'believe' shall be weeded out since, according to Leibniz, only 'terms' can be assigned symbols or numerical characters.

Leibniz's project of creating a calculus of thought and thus reducing reasoning to mere calculations has led to the advancement of computer technology in modern day. Although Leibniz had only conceived of a calculating machine, the algorithms derived from his binary systems have been regarded as the corner stone of programing languages. Truemper (2010: 2) adds on to say that Leibniz logical system, is to date, used to develop highly intelligent computer systems.

2.9 Boole's Logical Calculus

It is often remarked that Leibniz had made a notable attempt at actualizing his *ars characteristica* although he did not come to an entire realization of it; and that it only gained substantive development with George Boole. Leibniz's *characteristica universalis,* as Boole (1848: 11) opines, is not merely an exhibition of signs but rather, it is a language through which elements bound by mathematical laws are expressed. Casanave (2012: 89), states that Boole proposed his mathematical configuration of logic not merely as an instrument for solving problems in logic but also, as the structure underlying deductive reasoning. Signs and symbols, as Boole maintains, make possible all logical operations for they are governed by laws which specify the relations of propositions exhibited by characters.

Boole indicates that Leibniz's *calculus ratiocinator* distinctly expresses all mental operations involved in reasoning, and to exemplify this, he adopts Leibniz binary numeration. For Boole, 1 denotes 'universe' while 0 denotes 'nothing.' Like Leibniz, Boole articulates that the universe of all conceivable things flows from and is represented by and 1. In sketching his approach, Boole introduces the elective symbols: '*x, y, z,*' which represent the mental selection of conceptual subjects from a corresponding class; and to indicate the relation between objects, Boole allocates '+, –, ×' as operators and '=' as a symbol for identity (Boole, 1854: 27).

Paradigmatically, a class of X and Y represents all elements contained in each class respectively, and the mental process of selecting elements contained in X are denoted by x just as those contained in Y take the form y. Boole (1848: 2) indicates that the logical manifestation of the conceptual primes contained in class X, Y, and both X and Y take the mathematical form:

$$1x \text{ or } x$$

$$1y \text{ or } y$$

$$xy1 \text{ or } xy$$

Leibniz's calculus of thought dealt primarily with universal affirmative propositions; however Boole broadens it to include all categorical propositions as presented in Aristotle's dictum. Boole notes that negation as in 'not X' is expressed as $1 - x$; and particular propositions such as 'some X' are expressed as 'vx,' where the sign 'v,' according Antognazza (2018: 187) is juxtaposed to any elective symbol in order to highlight the specification of the given class. Thus, the categorical propositions, according to Boole's elective calculus, take the form:

$x = vy$	All Xs are Ys
$x = v'(1 - y)$	No Xs are Ys
$vx = vy$	Some Xs are Ys
$vx = v'(1 - y)$	Some Xs are not Ys

Boole stresses that all operations of the mind, in logical reasoning, are symbolized by the arithmetic operations of elective signs and as such, the connection between x and y is represented by the operator '+,' as in '$x + y$,' and that of multiplication is expressed as $x \times y$ *or* xy. Unlike Leibniz, Boole indicates that the manipulation of elective signs is governed by laws which forbid nonsensical operations such as 'x-y+y-x×xy=.' Boole notes that the peculiar nature of elective symbols confines them within the bounds of commutative, distributive and index laws.

In connection to this, he therefore writes that the commutative law is expressed as $xy = yx$; and distributive law is expressed as: $x(x + y) = xy + yx$; while the index law is expressed as $x^n = x$. Boole (1848: 3) argues that these laws, when taken as a whole, formulate a general axiom which underlies all reasoning and governs the form and presentation of mental operations. He emphasizes that if logical formulations and processes are to a great extent similar to those of algebra, then formal laws applicable to arithmetic would also apply to logical expressions (Boole, 1854: 31).

In arithmetic, the commutative law exemplifies how elements of a given set, combined by the operator '×,' can switch positions without altering their truth-value (Sidebotham 2003: 86). So given 3 and 9, $3 \times 9 = 9 \times 3$ will hold true given in both instances, the answer is 27. In Logic then, the expression $xy = yx$ holds. From this, Boole indicates that if xy express a whole wherein both x *and* y participate and signify something similar, it reasonable to deduce that $xy = x$; and this would imply that y denotes the same thing as x hence the equation would take the form $xx = x$. To avoid redundancy in arithmetic, Boole (1854: 31) notes that the expression xx would take the form x^2, therefore $x^2 = x$. This treatment is also applicable to the values of number where $1^2 =1$, and $0^2 = 0$ (Boole, 1854: 37). It follows from this then that $1x = x$, while $0x = 0$; and $1y = y$, while $0y = 0$.

Boole's algebra of concepts is not only a manifestation of a calculus but also an explication of the laws of thought regarded as axioms. Against the efforts of Leibniz and Boole, Hang (1960: 260) laments, "It seems as though logicians had worked with the fiction of man as a persistent and unimaginative beast who can only follow rules blindly, and then the fiction found its incarnation in the machine. Hence, the striving for inhuman exactness is not pointless, senseless, but gets direction and justification." Nonetheless, Siekmann and Wrightson (1983: 3) indicate that in light of Boole's publications, attempts to mechanize reasoning were advanced by Stanley Jevons who engineered a machine which, by pressing certain keys, it would display Boolean combinations.

2.10 Frege, Russell and Whitehead

Further advancements towards mechanizing logic, following the advent of the Boolean elective calculus, were presented by Frege, Russell and Whitehead. Frege criticized Boolean logic on account that it portrayed more of a calculus, and as such neglected the aspect of logic also being a language. Jacquette (2019: 127) opines that Boole's attempts only paved the way in the direction of algebraization of logic for he had not fully presented a combinatorial method in the Leibnizian sense. To resolve the defects of Boolean logic, Frege develops his 'concept script' which, according to him, is a language whose characters and formulas express content and thus capable of being determined as either true or false.

In light of Boolean variables, Frege introduced the content stoke '$\vdash$' which indicates that any proceeding symbolized proposition has judgeable content. In connection to this, Frege has been commended on his theory of quantification where particular and universal statements can be quantified as '$\exists x$' and '$\forall x$' respectively. Siekmann and Wrightson (1983: 4) say that Frege's *Begriffeechrift* contains a thorough explication of quantification theory. Despite the criticism directed to Frege's concept-script by the likes of Schroder, Siekmann and Wrightson (1983: 4) argue that, for the first time, the *Begriffeechrift* exhibited, in a concise manner, the vocabulary and syntax of Leibniz's universal language and its complementary calculus; and subsequently, underpinning computer programing languages and the varied contemporary mathematical logic systems.

Like Frege, Russell and Whitehead endorsed the thesis that all mathematical procedures can, in principle be reduced to logic, hence they deduced "a body of elementary mathematical theorems by explicit formal proofs" (Harrison, 2009: 473). The assumption here is that with logic as a basis for mathematics, mathematical theorems could be easily derived through computation. And for such theorems, proof procedures, which can easily be understood by minds which have been instructed with adequate background in symbolic logic, must be provided as justification for any such theorems (Whitehead and Russell, 1910: 3). By 'proof' Whitehead and Russell do not imply the representation of all processes involved in reasoning, but a deductive justification which determines the correctness or soundness of a theorem.

Wolfgang and Schmitt (2013: 78) submit that theorems provided by both Whitehead and Russell in *Principia Mathematica*, revived the need to mechanize reason and construct systems for developing theorems and machine-oriented prove checkers which would promote rigor in both logic and mathematics. Harrison (2009: 4) gives credit to Leibniz by arguing that he laid out prospects for automated reasoning by first outlining components of his *characteristica universalis*, calculus ratiocinator, and the feasibility of mechanizing mathematical reasoning.

2.11 Conclusion

The rationale behind mechanizing thought stems from the imperfections of natural language; and the hypothesis was that if logicians could succeed in developing a formal language underpinned by both a *characteristica universalis* and the *calculus ratiocinator*, all controversies would easily be resolved through calculations. For Leibniz, a creation of an axiomatic system, where all scientific knowledge can be univocally represented symbolically and subsequently, facilitating accurate computations was beyond doubt, necessary.

Nevertheless, there is more to natural language than the mere fact that it is plagued with ambiguities; and unlike automatic theorem provers, human beings are not machines whose thought patterns must, at all times, be governed by strict axioms put forth by the reasoning calculus. The essence of the problem lies not in the process of translating symbolic characters into natural language, but mainly with transcribing natural language into signs, characters or binary code. All things considered, Leibniz, when idealizing his project which aimed at mechanizing thought, failed to realize the discrepancy between humans and machines where it pertains reasoning and expressing thoughts.

Although Leibniz's *ars characteristica* has played a vital role in constructing programing languages and technology in general, it is not in accordance with humans' day-to-day reasoning routine. Language users use the very same imperfect language to think, express their thoughts, and transmit knowledge, with words and not symbolically. Hence, Leibniz's attempt to eliminate ambiguities in natural language for an error-free expression of thoughts is, in praxis, implausible.

Chapter Three: Automated Reasoning

3.1 Introduction

In line with the exposition of Leibniz attempt to mechanize thought, this chapter explores the conviction that, precision and exactitude can only be achieved through mathematizing natural language. It also highlights the process of arithmetizing language in order to eliminate errors in reasoning for automated reasoning. It attempts to bring to light how mechanizing both thought and language provides a framework for computing systems through which logical inferences could be made.

3.2 A Mathematical Framework for an Ideal Language

The importance of communicating precise and logical meanings in any given setting cannot be emphasized enough. A mathematical language, such as Leibniz's universal characteristic, attempts to eliminate semantic aspects of natural language so as to avoid the risk of indefiniteness. To achieve this, symbols are derived from the most basic concepts of ordinary language thus forming the alphabet of a perfect and logical mental language. From this standpoint, it seems quite reasonable to reject natural language as a means of communicating comprehensive meanings on the basis that it is pervaded by semantic indeterminacy which cannot to be symbolized logically since the presence of indefiniteness is thought to contaminate human thought processes.

Mathematical characters serve as both vocabulary and language for scientific discourse; and mathematical notation becomes a truth-purifying method for knowledge attainment. Factual claims about naturally occurring scientific phenomena can be arithmetized in order for them to be manipulated automatically as it is done in computer science. A mathematical language and the calculus of thought, in Leibnizian terms, are rather significant in the advancement of science. The application of logico-mathematical language to the business of scientific inquiry involves symbolizing natural language terms so as to translate them into formulae for computation.

As Paleo (2015: 9) observes, Leibniz's universal language and its complementary reasoning calculus have been most practical in the formation of algorithms for automated reasoners which are characterized as "conjecture checkers." Leibniz's idea of a symbolic language in pursuit of enforcing infallible reasoning, unambiguous communication of thoughts and a tool for discovery is incongruent with natural language since it contains ambiguity which often obscures meaning and thus leading to errors in reasoning.

The business of a mathematical language in science is not limited to outlining descriptive truths about the empirical (be it the real-world physical objects or the notions of the trans-empirical reality) but also, the concern of a mathematician is to manipulate signs and symbols in light of the clearly defined mechanical rules of the complex formal systems (Goldstein, 2005: 136). In the same vein, Ziman (1996: 18) argues that "Mathematical reasoning is immensely more powerful than plain language when it comes to generating verifiable predictions, unpalatable conclusions, or unsuspected connections between known facts." A mathematical language would seem most apt also for communicating with unmatched exactitude and definiteness. And it is for this reason that a logico-mathematical language and a rigid nature of mathematical notation.

3.2.1. Language Specification

The main goal of redesigning language was for the purpose of exactitude in communication such that informational content can be transmitted effectively from one agent to the other with minimal, if not zero errors or risks of misinterpretations. The process of disambiguating language and identifying the basic concepts is done through the language of clauses. Wos *et al* (1994: 3) indicate that the reason behind the formulation of the 'clause language paradigm' was for the sole purpose of proving theorems in mathematical logic; and for the most part, clause language has been the most apt generalizing and representing specifications about information.

Reducing natural language sentences to clauses results in an efficient language whose syntax is rather limited in order to erase any risks of ambiguity. Although the clause language seems to have restricted expressive powers due to the limitation of syntax, Jacquette (2006: 713) notes that its deficiency is not a liability but rather an asset for it helps avoid obstacles in the development of efficient systems of reasoning. Unlike natural language, whose vocabulary is rich and hence more prone to indefiniteness, the clause language limits its syntax and semantics but prioritizes precision.

As a way to discard irrelevant and redundant information so as to enhance the effectiveness of either human or automatic reasoners, natural language sentences are translated into clauses. Weiss (2012: 116) indicates that the only two logical connectives in clause language are 'or,' represented by ‘|,’ and ‘not,’ represented with ‘¬.’ In line with this, Jacquette (2006: 713) adds on to say that the logical connective ‘and’ is not explicitly stated between a pair of clauses but it is present. The following instances exemplify how clauses are retained from natural language propositions:

1. Socrates is mortal
   MORTAL(Socrates)
2. All humans are either male or female
   MALE(x) | FEMALE(x)
3. Socrates is not female
   ¬FEMALE (Socrates)
4. Socrates is human and male
   HUMAN(Socrates)
   MALE(Socrates)

Weiss (2012: 116) notes that the predicates in clause language are written in higher case letters while the sum is written in lowercase letters. Although an in-depth discussion of clause language is beyond the scope of this study, it must be indicated that clauses are not all there is to this language. In addition to the retention of clauses, Wos *et al* (1994: 3) point out that the language of clauses serves as a language of reasoning through which problems can be described, and in light of the strategies stipulated, rules of inference can be used to arrive at logical conclusions. Hutter and Stephan (2005; 205) assert that in comparison to natural language, the language of clauses may seem appropriate; it adequately facilitates the formulation of reasoning strategies and programs which not only effectively control reasoning but also, accurately provides means for deriving conclusions.

Since language is inextricably intertwined with reasoning, it is imperative therefore that, the language of reason be both precise and accurate to avoid misunderstandings and the possibility of error in reasoning. Clauses, therefore, in the language of clauses, create a logical language through which varied versions of formulae can be formed following predicate symbolism and use of constants and certain other functions (Vogt and Barta, 1997: 98). Since clauses, given they are natural language lexical items, may be vague or ambiguous, it is imperative then that they should be symbolized using logical symbols and combined together using logical connectives for precision.

3.2.2. Mathematizing Language

In light of the rejection of natural language as adequate, a means of unambiguous communication, a demand for a construction of an ideal language whose model based on the adoption of algebraic symbols and formulations gained more weight. The alphabet of an unambiguous formal language consists of formal symbols ($\forall$, $\exists$, x, $=$, y +, $\vee$, $\supset$, 0, ( ), &), analogous to the alphabets in natural language; and although unlike alphabets, the interpretation of formal symbols correspond to an entire word or phrase (Kleene, 1980: 69-70).

Governing these variables, are axioms which are a set of instructions specifying the juxtaposition between symbolic conjectures expressing statements of fact, which aids the translating of such facts into equations $\{(a \vee b) \supset q$, $\exists x(Bx \cdot Sx)$, $2xy- 3x^2y$, etc.$\}$ such that through calculus, they can be manipulated so as to determine the truth-value, and further discoveries. Ordinary language concepts are then to be translated into symbols and characters in order to attain precision. The manipulation of algebraic formulations through calculations then serves as a tool for building the body of knowledge through computing.

The variables and constants do not have a direct relation with objects in reality in a way that '*x* or 1' can be taken to be a substitute or representation of a real thing. The variables are not, in a strict sense, property of anything in the physical world since their meaning differs with different domains of applicability. For instance, the propositions, "there is a pig", "there is a pope" are to be formulated as '$\exists xP(x)$.' The declaration and consensus of what variables denote is thought to eliminate semantic indeterminacy.

However, in the language of clauses, quantification is not incorporated. Jacquette (2006: 711) notes that the universal quantifier 'for all,' expressed symbolically as '$\forall$' is removed and substituted with a variable that corresponds to it. In the language of clauses, the variable 'x' corresponds with the quantifier 'for all' and an automated program automatically interprets the variable as a substitution for quantification (Weiss, 2012: 117). The same goes for existential quantifiers whose representation is through clauses which specify the object referred to.

It would seem that an algebraic approach to language redeems natural language imperfections and as such, as a model of Leibniz's universal language, the *characteristica universalis* would indeed be an ideal language. However, Wurm (2021: 142) strongly argues that natural language cannot entirely be transcribed symbolically despite the logicians' attempts since it would be impossible to represent indefiniteness. Russell (1923: 86) concurs by noting that as unambiguous as the algebraic language would be, when human beings use it, it becomes clouded with ambiguity.

In polishing and perfecting language using this model, some information, which may be significant human communication, is filtered out given the restrictive nature of the algebraic model. Wurm (2021: 143) adds on to say that an arbitrary decision to ban ambiguity from language and thereby adopting a meta-language, does not eliminate ambiguity from language but rather, it suppresses it. When disambiguated, various meanings of a single proposition can be transcribed symbolically, and in this manner, is masked by the fact that each interpretation is symbolized unambiguously.

Weaver (2001: 3) stresses the fact that although algebraic formulations are precisely defined and exact, an absolutely unambiguous language is impossible. And even if it were hypothetically possible, Korner, (1962: 104-105) warns that "[t]he inexactness of the empirical and the exactness of the non-empirical concepts precludes isomorphism between the instances and relations of the two systems." Statements of fact about reality are not always distinctly definite and so is natural language. Algebraic symbols in logic, do not resolve semantic indeterminacy in a way that pragmatism in linguistics cannot, i.e., symbolism is for logic the same way pragmatism is for natural language.

3.3 Automation of Reasoning

The language of clauses, although precise and more favourable, does not serve as an appropriate medium for human communication, but nonetheless, it is a language used by numerous automated reasoning programs. Automation of reason, as Wos and Pieper (2000: 591) write, is act of programing computers using numeric and/or numeric representation so as to aid humans with problems that require extensive reasoning; and by reasoning, they do not imply approximate or probabilistic reasoning but logical reasoning, which necessitates that the conclusion derived must necessarily follow from the facts which it is derived from.

Moreover, Weiss (2012: 119-120) asserts that, in any case, when writing a program in clause language, one can express thoughts freely although with observance of the rules governing the replacement of clauses with constants or functions; and by so doing, one can have a reasoning program which will flawlessly reason and provide solutions to problems requiring mathematical computations and logical reasoning. The mechanical process which had been performed by the Turing machines was the realization of most mathematicians and logicians who had aimed at developing ‘logical computing machines’ (Yoos, 2016: 156).

Leibniz’s logico-mathamatical language is a language *understood* by computers and facilitates the communication between humans and computing machines. It is through the language of clauses that problems can be coded, and then decoded by an automaton which in turn automates the process of reasoning based on the available commands and as such, serves as a reasoning assistance in solving mathematical problems. Harrison (2007: 340) evidences this by indicating that most proof assistants since the late 1960’s were theorem provers used to determine the correctness of mathematical proofs undertaken by humans and later on, interactive automated reasoners were developed and worked together with humans in formalizing mathematical proofs.

In light of the fact that automated reasoners are semantics-blind, hence the need to translate language into a universal symbolic language, Leibniz’s idea of mathematizing language has found its realization in automated theorem provers and formalizing computers. Weiss (2012: 116) explains that it is not only impractical to write a computer program using natural language but is also not possible to use everyday language to simply direct a computer to execute certain tasks. To exemplify this, Paleo (2016: 7) identifies two proof assistants: Isabelle and Coq; and he notes that these two programs use simply-typed lambda calculus and are based on classical logic and intuitionistic logic respectively.

Computer programs are therefore written in a mathematical language which essentially contain a coded set of instructions understood by an automated reasoning machine. In explaining how the operational processes of automatic reasoners are carried out, Voronkov (: 1607) writes:

> Currently, theorem provers are used in the following way. The user specifies a problem by giving a set of axioms (a set of first order formulas or clauses) and a conjecture (again, a first-order formula or a set of clauses). If the input is given by first-order formulas, the prover should check whether the conjecture logically follows from the axioms. If the input is given by a set of clauses, the prover should check whether the set of clauses is inconsistent. In either case, for many applications it is desirable that the prover output a proof, if the logical consequence or inconsistency has been established. The proofs should either be human-readable (for example, when the provers are used for proving theorems in mathematics), or machine-checkable (for example, when provers are used as subsystems of proof assistants or verification systems).

Yoos (2016: 156-157) reveals that although the "Key to the simplicity of using logic and mathematical proofs on the database is the development of the logical languages," the complexity lies in mainly in the that automatic reasoning operations require not only complex but codified mathematical languages that humans need to be proficient in if ever they are to have automatic reasoning assistants. In so much as Leibniz had anticipated that should any problem or dispute arise, just so long as it can be translated into the *characteristica universalis*, it would be computed and resolved easily, certain problems that humans encounter, which are far beyond the scope of mathematics still remain resolvable by humans rather than being subjected to the complex operations of automation.

In the same vein, Harrison (2007: 334) articulates that at the moment "neither a systematic algorithmic approach nor a heuristic human-oriented approach is capable of proving a wide range of difficult mathematical theorems automatically. Besides, one might object that even if it were possible, it is hardly desirable to automate proofs that humans are incapable of developing themselves." To explicate the difficulty that follows having automated reasoners formalize proofs beyond human comprehension due to the complexity of a mathematical language, Paleo (2016: 7) demonstrates that automated reasoning systems often use cryptic languages which are usually machine-oriented and not human-readable to output proofs.

This occurs mainly because, despite the fact that automated reasoning programs could simply yield either a 'yes' or 'no', '1' or '0' answer as to whether a mathematical proof is sound or not, there is need for a detailed step-by-step outline of the reasoning or explanation behind the conclusion reached. In light of this, mathematicians and logicians who have mastered the ability to read binary computer language can, without any difficulty, comprehend proofs outputted by automatic reasoning assistants. It is evident that a mathematical language plays a pivotal role in logical reasoning and theorem proving, and for automated reasoning programs to compute proofs, they ought to operate within the bounds of explicitly defined rules of inference which serve as logical axioms for computing systems.

3.3.1. Rules of Inference and Algorithms

In order for an automated reasoning program to analyse, compute or draw conclusions from a given set of statements, it has to follows the logical conventions of reasoning which are known as inference rules. To infer simply means to draw conclusions based on available facts. And in

Automated reasoning, inference rules are rules of logical calculi which enable reasoning machines to formulate deductions in light of the information encoded on the database, and come to a conclusion based on facts. Vogt and Barta (1997: 100) explain that the rules of inference are also called logical algorithms.

Weiss (2012: 120) indicates that a set of various inference rules allow automated reasoning programs to reason in various distinct ways. To further illustrate this, Wos (1987: 201) demonstrates how the choice of an inference(s) rule generally impacts the performance of an automated reasoning machine. If an algorithm is incompatible with a computing machine, as opposed to one that's compatible, the result will inevitably be the failure of a reasoning program in addressing a problem or generating a valid proof (or none at all) of a given mathematical theorem. That is to say, different logical languages in automation use different algorithms and some algorithms are more efficient than others (Heaton, 2013: 71).

As Paleo (2016: 5) points out, Leibniz's attempt to solve the inconsistency and issue of plurality of meaning in natural language through the creation of universal language has been, to some extent, a success, which has however, resulted in the plurality of formal languages, thus creating an almost similar problem although at a more formal level. If then a chosen inference rule(s) is not congruous with the language representation understood by a reasoning machine, then the likelihood of an automated reasoned succeeding in either commutating or generating and checking the soundness of proofs is reduced to zero.

This goes on to show that the logical rules of calculi are not only complementary to a mathematized language for yielding sound conclusions in automated reasoning, but also that their correct application is imperative. The use of inference rules in mechanical computation guarantees logical soundness since it through them that automated reasoners and theorem provers can determine the relation between provided statements of fact and draw conclusions that follow with necessity. Some of the rules of inference used in automated reasoning are as follows:

1) Modus Ponens: if P is true and Q follows from P, it can be inferred that Q is also true,

$$\frac{\begin{array}{c}P\to Q\\ P\end{array}}{\therefore Q}$$

2) Modus Tollens: if P implies Q, the negation of Q implies the negation of P,

$$\frac{\begin{array}{c}P \rightarrow Q \\ \neg Q\end{array}}{\therefore \neg P}$$

3) Hypothetical Syllogism: if P implies Q, and Q implies R, it can be inferred that P implies R,

$$\frac{\begin{array}{c}P \rightarrow Q \\ Q \rightarrow R\end{array}}{\therefore P \rightarrow Q}$$

Employing inference rules enables automated reasoning machines to draw conclusions in a single step rather that the many required steps a problem or proof would require if these logical rules were not applied (Shostak, 2011: 317). However, Vogt and Barta (1997: 100) add on to say that in order for automated reasoning machines to execute commands, they need more powerful and complex rules in addition to the ones listed above, such as, binary resolution, unification, demodulation, and paramodulation.

For an automated reasoner to generate proof of a mathematical theorem and check the soundness of a given proof or try to solve any problem, the first step is to translate the problem from natural language into a mathematical language that can understood by a computer. And in line with the algorithm implemented, automatic reasoners or assistants perform computations which will lead to new conclusions based on the information provided by means of following a defined and definite strategy of reasoning. And in this way, reasoning becomes a mechanized process in which the possibility of errors in presentation and interpretation is eliminated by the exactitude of a mathematical language.

3.3.2. Applications of Automated Reasoning

Automated reasoning has found its applications in various domains of knowledge acquisition and experimentation. The two main applications of automated reasoning programs are in formal verification of mathematical theorems and in robotics and artificial intelligence.

3.3.2.1. Formal Verification

Given the limited nature of human mental capabilities in computation due to the limitation of memory, programmers often develop various programs and algorithms which can solve and provide automated argumentation of how the conclusion was reached. Most mathematical problems require computations which incorporate complex formulas whose structure consists of signs and numerous characters thus demanding the application of exhaustive rules which are beyond human mental capabilities. And for this reason, automated reasoning machines have been applied in mathematics, logic and computer science to function as reasoning assistants for human beings.

Automated reasoning programs are being used also for verification purposes thus serving as truth-purifying tools. Automated assistants are mainly used to detect inconsistencies in mathematical theorem proofs and correct any such errors in proofing. The application reasoning machines are not only limited to helping humans formalize proofs and check them, but also in automatic proof discovery. Ndungi and 'Uyun (2023: 484) write as follows:

> Automated argumentation is most commonly utilized in conjunction with deductive reasoning to locate, check, and verify mathematical theorems through the use of a computing system. When checking proofs with an automated reasoning system, the user can be certain that they have not committed an error in their computations.

In light of the fact that the language used to program automated reasoning machines is precise and the computational processes are also performed with clearly defined rules which, if unwaveringly adhered to in order to eliminate any possibility of error, this results in correct and reliable outputs. The objective of rigorously determining the accuracy of formal proofs has been forever attractive, but quite difficult for mathematicians and logicians, though simpler through automated argumentation. Hence it is most favourable to compute automatically rather than manually (Harrison, 2007: 342). Automated computations are the realization of Leibniz's dream to compute complicated mathematical problems using combinatorial methods while sparing human beings from labouring their days away trying to prove hard mathematical theorems.

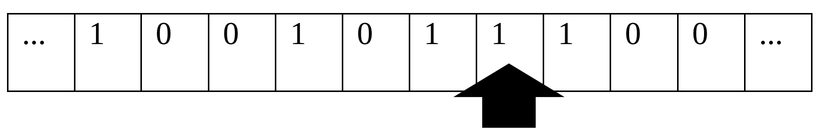
In the process of software verification, automated reasoning has been applied to identify bugs and improve the overall performance of computer programs. Voronkov (2003: 1609) concurs by noting that theorem provers are mandatorily tested for bugs, and this is done through extensive experimentation: following which if any bugs were identified, the development of debugging algorithms shall be facilitated in order to improve the efficiency and complexity of any such theorem provers.

Automated reasoning programs, in mathematics and logic, have been applied mainly to make computation of complex mathematical problems easier, and in so doing, applications have contributed immensely achieving Leibniz's dream of mechanizing both language and reasoning. Generally, Ndungi and 'Uyun (2023: 484) say that automated reasoning has been utilized to perform sound deductive process which include proving theorems mechanically and providing algorithmic argumentation and description of proofs.

3.3.2.1.1. Turing Machine

Leibniz had mentioned that if machines were to be used for calculations and formal proofs, men would no longer slave away their hours doing endless calculations. In his renewed publication *'On Computable Numbers, with an Application to the Entscheidungs Problem,'* Alan Turing introduced his idea of a computing machine known as the 'Turing machine.' Garzon (1989: 132) indicates that a Turing machine operates through codification of precise algorithms and as such, it can determine which tasks are capable of being successfully computed.

Turing and Copeland (2004: 8) describes that a Turing machine is made of a tape which bears infinite set of symbols, for instance 0s and 1s, each inserted in a square with a scanner, above or below, reading each square at a time. A pictorial illustration of a Turing machine is as follows:



When activated, the scanner reads symbols on the tape in either direction and can print the result in accordance with the instruction. The tape on the turning machine represents an algorithm resembling the Leibnizian binary system and also acts as a medium for inputting commands and outputting results. When set in motion, Turing machines operate in line with algorithms; and Turing (1948: 414) posits that any mathematical problem which can be mechanically computed by any human being can also, without failure, be computed by a Turing machine.

The only functions which Turing machine can solve, however, are computable functions. Turing, (1936: 321) says computable numbers are those which their expression as decimals is calculable by finite means. In a binary system, which is used in computing machines as a code, the base of 2^n is multiplied to a number and the result is a value of a real number whose calculation is by all means intelligible (Harrison,2009: 5). For example, given the number 98 whose decimal expression is 1100010, a Turing machine may be commanded to execute the calculation of the expressed sequences, hence Turning argues that it must be calculable and determinate as in:

$1100010 = 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

Similar to Leibniz's *characteristica universalis*, the language of the Turing machine is rigidly algorithmic and as such deterring input of ambiguous instructions which would subsequently lead to loopholes in the output to be generated. Turing (1936: 232) emphasizes that if ever a machine, in an axiomatic system, is configured equivocally, the moment it detects the ambiguity, it comes to a complete halt until an external operator resolves the ambiguity, and thus, resuming the assigned calculations. All non-algorithmic information must be symbolically expressed either as a program or a configuration so as to maintain precision.

Harel (1992: 233) asserts that for a given logical or mathematical theorem, if one were to adopt any programing language and develop an algorithm for the said problem, then code it on a turning machine, it would be solved without any difficulty. In computing, Turing (1936: 231) indicates that the behaviour of the Turing machine will be dependent on the given configuration expressed through the *m*-function: $f(q, x, y)$. So given a function E a Turing machine computes $E(x, y) = 1$, iff $x = y$, where $x \neq 0$, and $y > 0$; and $E(x,y) = 0$, iff $x \neq y$, where $x = 1$, and $y \leq 0$.

Turing indicates that there are varied versions of the Turing machine and Aberth (2001: 53) argues that the Turing machine, generally, is the ideal computing machine for it is both a *characteristica universalis* in that it contains arbitrary symbols as idealized by Leibniz, and a *calculus ratiocinator* since it also performs quick computations thus validating and invalidating proofs of configured theorems. Hang (1960: 260) contends that although it has not been necessary for mathematicians to learn predicate calculus, it is almost impossible for a machine to attempt a mathematical problem without first dealing with the underlying logic.

Paleo (2016: 9) refutes the claim that the Turing machine or any computing machine that has been developed since the technological era, can be regarded as ideal in the Leibnizian sense. Leibniz emphasized that the implementation of his *ars characteristica* would not only be a tool for judgment but also, will aid in discovering new knowledge about the world. Paleo argues that computing machines up to date are only "conjecture checkers" and not "theorem discoverers." Automated computing machines are pre-determined and follow algorithms, hence they cannot do something else other than what they are programmed to do. These *automated reasoners* (which are basically computing machines) are mostly confined to proving theorems either to be true or false, and thus merely exhibiting the application of the already existing knowledge rather than discovering entirely new theorems based on imputed axioms.

3.3.2.2. Artificial Intelligence and Robotics

In computer science, automated reasoning has found its application in the programmers' attempts to imitate human reasoning by creating artificially intelligent agents which have been endowed with the ability to reason autonomously and make decisions about *reality*. Harrison (2007: 342) says that the application of automated reasoning in artificial intelligence domains is geared mainly towards emulating human reasoning. The nature of reasoning in artificial intelligence agents makes use mainly of deductive inferences and it is for this reason that Kristensen (2021: 107) warns that human reasoning cannot be reduced to deductive inferences through axiomatic system, for there is more to the human mind than reasoning deductively.

Despite this, Al Kashari and Al Taheri (2019: 91) assert that the main essence of implementing automatic reasoning programs in artificial intelligence and robotics is to lay foundation for replicating human reasoning abilities and behaviour in an effective way. In order to attain these abilities, Heaton (2013: vi) notes that the algorithms are not just merely foundational but they are also useful in recognizing and processing patterns in a database since it is expressed through arithmetic calculations. Artificial intelligence algorithms resemble or mimic the structure of the human mind in order to attain the ability to use and understand language, reason and make inferences.

The ability of machines in the recognition and understanding of human use of language is an essential component for effective communication between humans and artificial intelligence agents and robots or amongst automated reasoning programs themselves. (Al Kashari and Al Taheri, 2019: 91). Gardent and Webber (2001: 489) highlight the fact that where it pertains natural language, there is need for algorithms which are capable of resolving semantic indeterminacy in order to filter out all irrelevant meanings so as to discern the meaning intended by a human communicating with a robot.

ChatGPT is a paradigmatic instance of an artificial intelligence system which has the ability to communicate with human users using natural language. This chatbot allows for humans to insert textual prompts which vary from instructions to either write an essay, an email, or simply generate interesting stories in natural language. This automatic reasoning program uses transformer neural links to learn and process language hence it is able to execute task prompts and produce results in natural language. Similar to the ChatGPT chatbot is My AI, although its availability is limited only to Snapchat users. My AI is artificial intelligence technology which has the ability to interact with users on a more social level in a manner that appears as though the user is communicating with another human.

Although the applications of automated reasoning programs has been successful in computer sciences ranging from numerous models of robots and artificial intelligence models, unlike theorem provers in mathematics, these agents are more susceptible to erring in light of the fact that they do not currently have an adequate grasp of natural language and its inherent complexities. For accuracy, natural language has to be narrowed down which in turn, makes it easier to create programing languages and algorithms capable of using natural language whilst minimalizing the possibility of misinterpretations and misunderstandings.

3.4 A Mathematical Discourse Disambiguation Strategy

Natural language, as noted previously, is rife with ambiguity, hence Leibniz deemed it an unbefitting language for science. For him, complex and ambiguous concepts of natural language ought to be translatable into mathematical formulae. In this way, precision and exactitude can be achieved thus making it easier to develop a body of scientific knowledge. Mathematics as an exact discipline whose truths are *a priori* and absolute, became a prototype of Leibniz's perfect language.

The fact that mathematics can pursue its truths independent on human experience, and establish such truths with absolute certainty, has rendered it a model which all attempts to attain knowledge had to aspire to; and as such, its standards of inquiry had to be mandatory and its methods applied if knowledge is to be attained at all (Goldstein, 2005: 121). Ambiguities, inconsistencies and contradictions cannot, in any way, be associated with mathematics, and in light of this, it seemed reasonable for Leibniz to arithmetize language so as to eliminate any instances of indefiniteness.

Leibniz's thirst for precision and certainty epitomized mathematical certainty, and to attain this ideal, algebraic formulations would become the language of science. Although it may seem desirable to use a mathematical language for the sake of logical precision as Ziman (1996: 14) indicates, reducing natural language in natural discourse between humans would simply censor meaning to misleading triviality. Advancements of Leibniz's idea of a logical system that seeks to replace human reasoning with mathematical notation seems rather superfluous since a human is not purely deductive, but rather, it can be probabilistic and approximate. To show the inadequacy of attempts to formalize natural language for human-to-human communication, Feyerabend (1993: 11-14) elucidates that expressions of thoughts are not merely constituted by conclusions or facts derived from intuitively true facts, but rather, they also consist of errors in interpretation of facts, and imperfections resulting from conflicting interpretations.

3.5 Conclusion

In general, the ideas of translating mathematical language problems into Leibniz's *characteristica universalis* and solving them through logical calculi with the assistance of automated reasoners and theorem provers has led to the advancement of the sciences, in particular, the hard sciences. However, it must be pointed out the mathematical rigor and precision of Leibniz's mathematical language has its limitations clearly defined within the bounds of mathematics and computation. Despite this, Leibniz's universal characteristic has been able to do away with ambiguity thus paving way for unambiguous communication amongst logicians and mathematics. Nonetheless, as it shall be shown in the following chapter, a mathematical language does not eliminate ambiguities in normal everyday discourse settings where expressions of thought are carried out using ordinary language.

Chapter Four: The Peculiar Nature of Natural Language

4.1 Introduction

The main focus of this chapter is to give an analysis of indefiniteness as a characterizing feature of natural language, which, according to logical orthodoxy advocated for by Leibniz, and as discussed in the previous chapter, is a reprehensible excrescence defining the imperfection of natural language. Contrary to this conception, this chapter explores semantic indeterminacy in-depth for a better appreciation of it, highlights the pragmatic mechanisms employed in order to resolve ambiguity in natural language, and the significance of natural language in the communication of substantial thoughts about reality.

4.2 Semantic Indeterminacy

A shared assumption amongst logicians is that the imprecision of natural language leads to uncertainty of interpretation, and also of information in general. Natural language propositions often oscillate between what a speaker means in making an utterance and what the terms of an utterance mean, thus allowing for numerous interpretations to be deduced. The resulting indeterminacy of meaning results in the classification of ordinary language statements as being either true or false impractical given the obscure meaning specification of terms.

In describing a given state of affairs, an assertion may provide informational content whose readings cannot be assigned a definite truth-value of either truthfulness or falsity. However, such an assertion does not necessarily become nonsensical as logicians would render it to be. Although it may prove to neither be definitely true or false, it may be true or false partially, that is, when contextualized, it can be true in one situational setting and be false in another. The absence of absolute certainty in regard to the meaning natural language propositions presents its peculiarity.

The semantic indefiniteness of propositions ought to be acknowledged and embraced as an inescapable yet resolvable element of language, rather than something to be eliminated, as Leibniz had aimed at following the advancement of his *characteristica universalis* and *calculus ratiocinator*. Byers (2011: 82) declares that semantic indeterminacy cannot be avoided in an absolute sense; and that although it is presumed to delay the quest for knowledge, it can be resolved and thereby making it possible for the re-evaluation of the truth of a proposition which may have been previously declared undetermined. The recurring indeterminacy of propositions in ordinary language is mainly due to the creative use of language perpetuated by ambiguities and vagueness or even use of non-factual aspects of language such as irony, sarcasm, humour and so on.

4.2.1. Primary Senses of Indefiniteness

Natural language, in itself, is not indeterminate: the indeterminacy of meaning is owned solely to lexical items of a language whose presence in an utterance results in the imprecision of propositions, thus rendering their truth-value undetermined. Wittgenstein (1958: 80) affirms that the "meaning of a word lies in its use." Words in ordinary language can be used to formulate ambiguous or vague propositions but it does not necessarily follow from this that natural language is, in itself, imprecise.

Obscurity of meaning is owed mainly to the usage of words since, when used in a specific context, the meaning of a word becomes clear and yet in another, it becomes unclear. Subsequently, those lacking a substantial grasp of language make haste in labelling natural language a labyrinth that one ought to escape. Ambiguity and vagueness are the two main aspects of language which, if not properly decoded, can result in misinterpretation and misunderstanding of information.

4.2.2. Ambiguity

Ambiguity is a linguistic phenomenon where a single word is assigned multiple meanings and whose application makes varied readings or interpretations plausible. To a great extent, almost all words of a language express more than one sense of meaning when uttered out of context. Although this inherent nature of language may be aesthetically pleasing, Leibniz and his followers found ambiguities to be anything but pleasing, hence their notable efforts in trying to eliminate the said flaw from language entirely.

Whenever ambiguity is present in a statement, it is mainly because the word or the informational content of a given proposition is not explicitly defined. Byers (2011: 86) rightly points out that there are two or more ways of looking at an ambiguous situation and each point of view is always incompatible with the other. The nature of ambiguity presents a *this* or *that* alternative but never both. The meaning can either be apprehended as either one of the conflicting possible meaning or neither, and this is resolved by disambiguation.

The meaning of an ambiguous expression cannot be, at the same time, taken to imply both or all senses of what is expressed. It is the failure to grasp the nature of ambiguity that led Leibniz and his counterparts to classify ambiguity as a defect whose remedy necessitated a fabrication of an ideal language. One cannot determine the meaning of an ambiguous statement if they lack the necessary knowledge about language and the linguistic capability to determine the potentially correct interpretation of any ambiguous phrase.

The inconsistency of meaning in ambiguous terms and the process of disambiguation is considered insufferable by the logician. For him, all thoughts contributing towards the attainment of knowledge must necessarily be unambiguous and communicated in the same manner, as though reality, in itself, is effortlessly unambiguous. Ambiguity for Leibniz is rather disagreeable, and the following proposition is paradigmatic of the aggravation it caused for him and his collaborators.

Consider the ambiguous phrase, 'I saw a soldier on a battlefield with some binoculars.'

Plausible readings of the proposition are as follows:

a. 'I saw a soldier on a battlefield through some binoculars.'
b. 'I saw a soldier on a battlefield and he had some binoculars.'
c. 'I was on the battlefield and saw a soldier through some binoculars.'
d. 'I saw a soldier standing a battlefield, and on the battlefield, there were some binoculars.'

The uncertainty of the correct meaning of the initial proposition is what made Leibniz uncomfortable. This is because, if the proposition meant the interpretation expressed in (a), then it would not mean any of the subsequent interpretations, and if it meant what is stipulated in (d) likewise, it would neither mean any of the preceding interpretations. Pinkal (1995: 81) asserts that the burden of correctly determining the expressed sentential meaning can be relieved through seeking clarity since the ambiguity is often perpetuated merely by inadequate understanding.

Leibniz erroneously portrayed ambiguity as though it is something permanent which forever plagues natural language and failed to acknowledge that it is something that occurs only when language users use language in one way and not the other. Words of a language, in their own right, are not ambiguous, but can be used ambiguously, either intentionally or unintentionally. Ambiguity in language, as Byers (2011: 88) advances, is a dynamic happening and not something that is eternally fixed. Its occurrence is solely dependent on the use of language.

The numerous senses of meaning in an ambiguous phrase, which make it appear as though it would be impossible to discern the precise sentential reading, ground the assumption that natural language, in its entirety, is unsystematic and therefore logically imperfect. In the same vein as Leibniz, Carnap (1937: 2) argues that ambiguities explicate the complex nature of natural language and as such, making evident the idea that formalization would hardly ever be feasible, thus necessitating arithmetization of syntax. The complexity of language here is frowned upon as though the reality it describes is effortlessly simple when it is in fact, just as complicated.

Contrary to Leibniz, Boole, Frege, and the likes, Byers (2011: 86) explains that the incompatibility of meaning in ambiguous propositions must be viewed as an essential element to be cultivated rather than an enemy to be eliminated. He further notes that there are two approaches to ambiguity: the first being that one can resolve ambiguity by accepting one point of view as correct and the other as incorrect; or one can view ambiguity as providing an opportunity for creativity and in-depth understanding of a given state of affairs.

The use of language warrants the presence of ambiguities and despite all attempts of symbolism to make language unambiguous, so as to depict the exact processes of the mind, it is rather not possible to eliminate ambiguities because the responses of the mind to reality cannot be unambiguous since the mind is not a digital computer (Bronowski, 2008: ch. 5). A generally held contention amongst logicians is that ambiguities are blameworthy for deception and fallacious reasoning. However, what is questionable is their overambitious attempts to erase ambiguities from natural language in the name of precision.

It was bewildering for Leibniz that ambiguity is immensely persistent in language, hence, he attempted to put forward a scheme aimed at getting rid of it. This is made evident by the substantial efforts to construct and further develop a *characteristica universal* — an unambiguous language embodying both exactness and precision — and a *calculus ratiocinator.* At odds with efforts in this direction, Lobeck and Denham (2006: 229) caution that human language is not, in any sense, mathematical. Algebratizing language and mechanizing thought out of consideration for precision results in an exclusive configuration which is unbefitting for human expression, but apt for droids or their machine counterparts.

Bearing this in mind, Winkler (2015: 47) explains that it is not surprising that after attempts made to erase ambiguity from natural language, there is not much, if any, evidence stipulating the decrease or absence of ambiguities in human language. Notwithstanding such efforts, if knowledge about reality is to have any substance at all, it is necessary to have an appreciation of natural language and its inherent aspects in lieu of a mathematically universal language divorced from reality as a whole and thus serving as a perfect dialect for human communication.

To resolve ambiguities, it is essential to possess adequate linguistic knowledge needed to master the analysis of sentences. Ambiguities are divided into various subtypes howbeit only two shall be discussed here, namely: lexical ambiguity and structural ambiguity. Lexical ambiguity refers to the kind of ambiguity that arises from difference in meaning of a specified lexical item. This kind of ambiguity is also known as semantic ambiguity because the word ‘semantic’ places emphasis on the fact that the ambiguity lies within a word other than the grammatical structure of a given proposition.

Keith (1986: 147) indicates there are two kinds of lexical ambiguity namely: Polysemy and homophony. In order to distinguish these two classes of semantic ambiguity, Saeed (2003: 6) explains that in lexicography, polysemous senses of ambiguity are listed and treated under the same lexical entry while homophonous senses, on the other hand, are treated as separate entries. Polysemous ambiguity occurs when a word has different meanings while homophonous ambiguity occurs mostly in spoken language, when words have different meanings and yet similar pronunciation.

Take for example: ‘Evie saw a bat’

The above sentence exemplifies a polysemous ambiguity contained in the words ‘saw’ and ‘bat’ where ‘saw’ can be interpreted as either an act of perceiving or as an act of cutting with a saw; and where bat can mean either a mammal or a tool for baseball. In this case, the exact meaning of the statement can be made clear through additional information or, as in most cases, taking into consideration the context surrounding the assertion.

Consider the sentence: ‘He took a bow/bough’

In this case, the ambiguity arises from the two homophones. This is not usually the case with written language since the spelling of the word immediately specifies the sense in which a homophone is used. However, in spoken language, the sense in which a homophone is used may make it difficult to decipher exact meaning of the term thus resulting in a case of ambiguity. In light of the above examples, given the nature of words and their meanings, Cuacuh (2012: 232) concurs that strictly speaking, lexical ambiguity is an inherent attribute of linguistic expressions.

Structural ambiguities, on the other hand, arise when a phrasal expression has numerous syntactic representations, and as such, admitting to more than one interpretation. Unlike lexical ambiguities which arise from the vocabulary of a language, structural ambiguities occur mainly due to the underlying structure of a given proposition. Dick (1995: 72) claims that structural ambiguities come about as a result of careless use of words in the process of sentences formation. Konopka (2007: 156) denounces Dick's claim by pointing out that structural ambiguity in natural language is not a pathological occurrence resulting from careless composition of propositions since oftentimes, ambiguities are unintentional and nonobvious.

In the same vein, Kreeft and Dougherty (2010: 47) add on to say that natural language users are not aware of the fact that certain combinations of terms in a phrase can beget structural ambiguities unless one intentionally appeals to ambiguity for making puns or in an attempt to avoid commitment to what is expressed by a proposition. Structural ambiguities are present in instances where language users produce incomplete sentences in an attempt to avoid redundancy, and also, in propositions which do not have formal signals responsible for limiting syntactic interpretations to one possible reading.

Take for example: 'Evie loves chocolate more than her brother'

Upon analysing this proposition, one becomes aware that not one of the words in this statement is ambiguous but rather, that it is the underlying structure of proposition which makes it open to more than one interpretation. Possible interpretations could be that 'Evie loves chocolate more than she loves her brother' or 'Evie loves chocolate more than her brother loves chocolate.' In an attempt to avoid the superfluous repetition of the terms 'loves' and 'chocolate,' a natural language speaker would choose to withhold the very same information that makes the proposition precise with the assumption that it would be no difficulty to decode the intended meaning.

Another example: 'The school admits intelligent boys and girls'

The above sentence, in a similar manner as the previous sentence, is open to two interpretations because of more than one constituent structure. The proposition above can either be interpreted to mean that 'the school admits both intelligent boys and intelligent girls' or 'the school admits only intelligent boys and girls (all girls, intelligent or not).' It is not obvious as to whether the adjective 'intelligent' modifies the noun 'boys' and not the noun 'girls;' or whether it modifies both nouns simultaneously, hence the ambiguity. The above sentences explicating instances of structural ambiguity do not merely resemble careless use of language as Dick (1995: 72) alleges. Taha (1983: 251) assumes that all ambiguous sentences are ungrammatical for they lack formal signals such as stress, juncture or punctuation, affixes, pitch, inflections, major class membership and function words. The point worth noting, however, is that structural ambiguity does not arise when words are used haphazardly without direction, but that ambiguity can arise in grammatically correct sentences as shown above.

In rejection of Taha's claim, it is necessary to submit that ambiguity simply denotes a syntactic mapping open to more than one semantic reading, which is both grammatical and meaningful respectively; and if ambiguous propositions are ill-formed as Dick assumes, then they would be ungrammatical without any semantic content. But this is not true of ambiguous propositions. The problem here lies with Taha's treatment of 'grammatical' and 'meaningful' as though they are synonymous terms. Both readings of an ambiguous proposition are grammatical and meaningful just as it is the case with unambiguous propositions.

Generally speaking, Simatupang (2007: 103) argues that in most cases, natural language users do not know or are not even aware as to whether propositions convey with precision the intended meaning or whether they contain an ambiguity. He further notes that it would require linguistic proficiency for one determine the presence of ambiguity, avoid or resolve it if possible. Despite the perverse nature of ambiguity in natural language, users use language effectively without any insurmountable difficulty.

Ambiguity cannot be divorced from language because it is constitutive of the very act of communicating thoughts, that is, if thoughts are to be communicated at all, and language to be used, there will always be room for ambiguity regardless of all attempts made to avoid or eliminate it from natural language. In opposition to Leibniz's submission which portrays ambiguity as a defect in language and thus opting for algebraic symbols as what would constitute what he deemed a perfect language, Konopka (2007: 156) reveals that the unambiguous nature of the Leibnizian *characteristica universalis* is oddly restrictive for human expression in natural settings since identifying symbols or characters as lexical items results in algorithmic formulae which is inadequate of capturing entirely all aspects of natural language.

Similarly, Medina (2005: 53) argues that the acclaimed superiority of reducing natural language to arithmetization is not quite explicit since mathematical notation is suitable for calculations and not apt for many other functions performed by natural language despite its indefiniteness. Ambiguity is an indispensable property of language and Zhang (2007: 191) evidences this by saying that the multiple senses of meaning together with flexible structures generated by syntax make ambiguity an essential aspect of language.

Despite ambiguity being an uncomfortable property of language for Leibniz and other logicians, it cannot, in praxis, be entirely eliminated for it is embedded within natural language and as such, it should be tolerated as an inescapable aspect of language. Closely related to ambiguity is the concept of vagueness, which likewise, possesses the problem of interpretative uncertainty, and as such, taken as another primary instance of semantic indefiniteness.

4.2.3. Vagueness

Natural language users tend to communicate effectively using vague expressions, which, in most cases, interlocutors are not conscious of; and even if they were aware of the vagueness, the Leibnizian qualm that demands precision through mathematical analysis for the purposes of applying logic to propositions is never raised. Vagueness then, like ambiguity, is an integral feature of language. Where it pertains defining vagueness, from a linguistic point of view, Burns (1991: 3) notes that there are various conceptions of vagueness such that it is unclear as to what the commonality amongst the proffered definitions is, and this makes it difficult to define vagueness.

Concurring with Burns, Ullmann (1962: 118) and Austin (1962: 125) attest that provided one undertakes a detailed analysis into the nature of vagueness, one will soon realize that the term ‘vague’ is, in itself, rather vague. Fine (1975: 265-266) roughly defines vagueness as lack of meaning, and further notes that any expression capable of expressing meaning is also likely to be vague. In a similar manner as lexical ambiguities, vagueness is perpetuated by lexical items and in certain cases, it is possible to have an expression that is both vague and ambiguous.

One element that distinguishes vagueness from ambiguity is that, in logic, it is often attributed to predicates and it often leads to paradoxes while this, on the other hand, is not the case with ambiguity. Vagueness has been associated with borderline cases and the sorites paradox (Raffman, 2014: 18). Given the presence of a vague predicate such as 'tall', it would seem rather impossible to determine with precision its scope of applicability and therefore, this inevitably gives rise to borderline cases. The sorites paradox resulting from borderline cases illustrates the importance of vagueness in natural language for it exemplifies just how difficult it is for the human mind to cognize with precision certain given states of affairs.

The presence of borderline cases is what characterizes vagueness such that if a predicate has no borderline cases, it is prima facie clear. The sorites paradox, which paradigmatically arise from vagueness, is infamously demonstrated through the problem of the heap. Employing an interrogatory form to exemplify the sorites paradox, Williamson (1994: 8) asks "Does one grain of sand make a heap? Do two grains of sand make a heap? Do three grains of sand make a heap? Four? Five? … Do ten thousand grains of sand make a heap?"

For the sake of practicality, one would be more inclined to respond to the first question in the negative because one grain of sand is evidently not a heap. Following this line of argument then, it seems only plausible to respond to the subsequent questions negatively since adding one grain of sand to a non-heap cannot transform it into a heap. Therefore, ten thousand sand grains, in a similar way, cannot constitute a heap. The argument is as follows:

I. (P_1) One grain of sand is not a heap

(P_2) If one grain of sand is not a heap, then adding one grain of sand to a non-heap does not result in a heap

∴ A pile of ten thousand grains of sand is not a heap.

The paradox serves as substantial ground for logicians to denounce natural language for logical purposes: despite the impeccable reasoning process leading to a valid argument, it is possible for one to deduce a false conclusion from true predicates. Presented formally, according to Schmitz *et al.*, (2011: 127), where S_n denotes the n grains of sand collectively, the argument takes the form:

II. (P_1) $\neg(\text{heap}(S_1))$

(P_2) $\forall n[\neg(\text{heap}(S_n) \rightarrow \text{heap}(S_{n+1})]$

$\therefore$ $\forall n[\neg\ \text{heap}(S_n)]$

In ordinary language— English in this case— the definition of 'heap' implies a creating of a pile by means of stacking things, one on top of the other, in a structured manner. Without any deliberation, it is quite explicit that the first premise is true. But to conclude that no pile of sand is a heap, is a falsehood. The paradox surfaces mainly because there is no exactly defined boundary which determines with precision the applicability of a vague predicate, i.e., it is not entirely clear at what point exactly does a heap become a heap or cease to become a heap. A stack of nine grains of sand cannot be said to be a heap as it would be deemed as too small; but it is unclear as to how many more sand grains would suffice for a classification of heap. It would seem favourable for logicians to commit a numerical value of grains of sand which demarcate a heap from non-heap; but to designate a definite point where the heapness and non-heapness of a heap are precisely defined would be absurd since heaps are made of sand grains piled up in a random fashion.

The problem of sorities is a logical problem: a self-imposed predicament resulting from an eagerness to attain absolute precision and perfection. Such problems, if they occur at all, in natural language, are neither normally insignificant nor are they regarded as problems in need of a solution (Williamson, 1994: 72). The idea of applying logic to indefinite propositions is quite unsettling and causes anxiety for most logicians. Indeterminacy in natural language is a fatal flaw in logic and its scope of application for Leibniz; hence his universal language and the reasoning calculus aimed at erasing the claimed defects from language entirely.

However, in his analysis of natural language, Russell (1923: 86) argues that vagueness is inevitable, and this is evidenced by the that fact logical propositions become vague in the face of evaluation because of the vagueness of the terms 'truth' and 'falsity.' Leibniz and his collaborators uphold an idea of a pseudo-absolutist precision which all propositions are subjected to if ever their truth-value is to be determined, and yet the communication of facts, whose medium of exchange is natural language, statements about reality are not, strictly speaking, straight forward and in conformity with either 1 or 0.

In his analysis, Hempel (1939: 163) disregards the pernickety of logicians in refining natural language concepts in view of the fact that generally speaking, human knowledge is not absolutely precise, but rather, like natural language, though to a lesser degree, also exhibits certain levels indefiniteness and uncertainty. It is effortless to fantasize about a universal language whose vocabulary is constituted by precise symbolism, as it was the case with Leibniz, but to construct such a language is not feasible (Russell, 1923: 86). Russell also observes that logical symbols, however precise they may be, when used by humans, become vague.

Vagueness then, is not to be taken as characterizing the imperfection of language since much of human knowledge is inexact hence it would be unreasonable to demand that the terms expressing knowledge to be exact (Kreeft and Dougherty, 2010: 48). It is erroneous to stigmatize vagueness as a semantic defect to be eliminated from ordinary language. And inasmuch as Leibniz's mathematics language was intended to express scientific facts with utmost precision, Kosko, (1994: 8) maintains that propositions expressing facts about reality are not exclusively either true or false since in actual fact, their truth value lies somewhere between absolute truthiness and absolute falsehood. He further notes that statements of fact about reality are not only tentative, but also, like natural language propositions, they are vague and imprecise.

Deemter (2010: 113) says that although naturally occurring languages embody a reputation of messiness, they are partially systematic. The tidiness promised by formalization has its use restricted to mathematicians and computer scientists thus serve no purpose in communicating thoughts in any natural discourse. In support of this, Russell (1923: 84) asserts that despite his efforts in advancing the creation a formal language in Leibnizian terms as an attempt to escape vagueness, it is unfortunately a private language, ill-suited for expressing thoughts in public settings.

Because vague terms are likely to result in paradoxes and puzzles, the basic assumption has always been that if scientific propositions were to be represented arithmetically and embodying mathematical precision, then their truth would be determined; and therefore, making plausible the attainment of knowledge. This postulation presumes reality to be entirely definite. Nonetheless, physics and quantum logics have proven this to be but counterfactual. Statements expressing factual content about natural phenomena are not wholly true or wholly false.

The sorites paradox makes this contention evident since, given a spectrum, one ought to mark with precision the transition point; and yet this has proven to be a difficult task which shews that the persnicketies sought after by logicians serves no purpose in natural discourses. Leibniz had anticipated the dangers of accepting indefiniteness as an aspect of both language and reality: given the extreme ends— truth and falsity— vagueness allows the presence of a mid-point which balances judgment, where valuation can be either one of the truth-values or both. Lack of symmetry in vagueness invalidates two fundamental principles: the principle of non-contradiction and the principle of the excluded middle.

To exemplify the rebellious nature of vagueness, let us take for example the proposition "All women are beautiful." 'Beauty' is vague predicate and also, 'women' is used rather generally to adult females, hence it is not specific as to which woman exactly is being referred to— it generalizes without any specifying details. Assigning symbols as substitute for indefinite terms does not result in precision as Leibniz has idealized; and Peirce (2011: 295) evidences this by saying that signs can be objectively indefinite inasmuch as their interpretation is to a great extent indeterminate, and as such leaving the burden of determination on the interpreter's abilities.

Provided the generality presented by the vague proposition, Peirce (2011: 295) points out that if the proposition is transcribed symbolically to eliminate the indefiniteness, as in 'R is P,' the principle of excluded middle shall be applicable but will not hold at all. Given the form of the principle of excluded middle, 'p v ¬p,' it does not hold true for vague propositions since valuation cannot be only limited to 'R is P, or 'R is not P.' It is not the case that women are either beautiful or not beautiful since beauty is an indefinite term and subsequently, it does not conform to the categorical dichotomy of two-valued logic.

In connection to this, given the nature of vague predicates, where the categorical division between mutually exclusive values of true and falsity allows the presence of otherness or spectrums of possibility, the principle of non-contradiction fails to apply in the sense of it being true. "The *vague* might be defined as that to which the principle of contradiction does not apply" (Peirce, 2011: 295). The principle of non-contradiction is true of definite propositions where an expression of complementary opposites cannot be both true and both false in the same respect: if 'R is P' then 'R is not P' is false and vice versa.

In contrast, borderline cases make it possible to conceive of a third state from which we can oscillate between two interpretations of a proposition and in which truth and falsity are not mutually exclusive. Vagueness marks the mid-point between black and white, 1 and 0, truth and falsehood. It is at this point where, in referring back to the sorites paradox, a heap can both be a heap and a non-heap simultaneously, as an established truth. Annoni (2006: 102) summarizes this by saying that the principle of excluded middle and the principle of non-contradiction can only apply to definite terms and it is only such instances that the principles can be thought to be applicable, but that their applicability does not translate in their being absolutely true since there are cases where they are applicable but false, as in the case of vague expressions.

Logicians and mathematicians opted to either ignore vagueness or eliminate it as in the case of Leibniz. Instead of appreciating vagueness as a fundamental resource and not a nuisance hindering the attainment of knowledge, he went ahead and attempted to create a *characteristica universalis;* as an ideal language which is in conformity with the laws of logic and discarded natural language assuming that by so doing, its imperfections would be dealt away with.

It is unreasonable to act as though one were an ostrich and bury one’s head in the sand by fabricating languages in pursuit for perfection and hoping flaws about natural language will disappear; rather, it seems reasonable to embrace a logic of vagueness— a logic modelled on three truth-values: truth, false and indeterminate— in which the sorites paradox may be situated in the world of reality and not the Leibnizian world of desired yet unattainable precision and abstract calculations.

4.3 Philosophical Principles Governing Discourse

In the previous section, it has been explicated just how the presence of ambiguity and vagueness perpetuate obscurity of meaning in language, and to address this from a pragmatic point of view, Grice (1975), in his renowned paper “*Logic and conversation*” provides a framework for purposeful and effective communication. Hadi (2013: 69) explains that for Grice, communication must at all times adhere to the cooperative principle based on the assumption that, when interlocutors are involved in a discourse, they cooperate or collaborate for the sake of meaningful conversations.

The cooperative principle, according to Grice (1975: 45) is as follows: “Make your conversational contribution such as is required, at the stage at which is occurs, by the accepted purpose or direction of the talk exchange in which you are engaged.” This principle holds for any interactions thus necessitating their observation at all times, and if one opts out of the principle, whatever they utter shall be deemed nonsensical. In the same vein, he develops the following maxims:

- The Maxim of Quantity: Make your contribution as informative as is required
  Do not make your contribution more informative than is required
- The maxim of Quality: Do not say what you believe to be false
  Do not say that for which you lack adequate evidence
- The maxim of Relation: Be relevant
- The maxim of Manner: Avoid obscurity of expression
  Avoid ambiguity,
  Be brief (avoid unnecessary prolixity)
  Be orderly Grice

(Grice, 1975:45-46).

The main idea behind the cooperative principle and the maxims is that, when interlocutors converse, they do so in a logical and rational way and as such, there is some cooperation at play. The maxims serve as imperative instructions which one ought to align oneself with for the sake of engaging in rational, logical and successful communication of thoughts (Dynel, 2018: 35). The cooperative principle and the four maxims set explicitly the standard behaviour which ought to be part of any meaningful discourse.

Of interest to us is the maxim of manner. As stated, Grice prescribes that when articulating thoughts to other people, one ought to be precise and subsequently, avoid obscurity of meaning. Ladegaard (2009: 651) argues that Grice's own definition of 'cooperative,' in cooperative principle, is vague and ambiguous, and as such, have enabled people to use varied interpretations of the term for both the purposes of advancing the understanding of the principle and also, for criticizing it altogether. Thomas (1998: 176) identifies that only a handful of Grice's followers have noticed the ambiguity with Grice's cooperative principle, while most failed to recognize it; and those who have, have made efforts to disambiguate the ambiguity of the word 'cooperative.'

This exemplifies the inescapable nature of ambiguity. Davies (2000: 7) explains that the ambiguity in Grice's cooperative principle lies between the technical and the non-technical sense of the word 'cooperative' and he goes on to demonstrate the confusion caused by the ambiguity. The very act of prescribing how human beings ought to express their thoughts in a restricted and controlled manner takes for granted the fact that the manner in which human beings communicate is not always straightforward. Humans can *try* to be as precise as they can be, but this does not get rid of the fact that imprecision is an integral component of language.

Leibniz erroneously identifies humans to be in close relation with machines hence he apprehends human beings' thinking and communicative processes to be exclusively based on rationality and logic. To adequately grasp the meaning of what is or has been said in a given setting, it is true that logic and rationality play an essential role, but most importantly, the context within which the discourse takes place and background knowledge shared by both speakers enable interlocutors to have a meaningful understanding of what is being communicated.

Leibniz focuses more on the semantics of natural language and paid no attention to the linguistic knowledge which natural language speakers utilize so as to attain a meaningful interpretation and understanding of what propositions mean. At most, Grice is reasonable enough to observe that human beings may, unintentionally or with intention, decide to either violate, opt out, infringe or flout a maxim; and thereby creating implicatures (Grice, 1975: 49).

In a communicative discourse, one may flout a maxim for the purposes of being ironic or humorous; and this are some of the aspects of language which Leibniz disregards as altogether imperfections of natural language. Contrary to Leibniz's perspective, Lockyer and Pickering (2019: 7) argue that these features are inseparable aspects of language and as such cannot be isolated from language, but are to be appreciated as part of the general spectrum of communication.

4.4 Secondary Senses of Indefiniteness

The presence of ambiguity or vagueness in natural language propositions is one of the reasons why human communication results in the non-observance or flouting of Grice's maxims. Ambiguity and obscurity of meaning can lead to other kinds of indeterminacy where language is not used in a definite sense. For one to comprehend the meaning of terms used ambiguously or in an obscure manner in a proposition, one appeals to different cognitive process, whether they are conscious of the process or not. Among others, humour, and irony and sarcasm, are subtypes of indefiniteness where obscurity or ambiguous terms may be used for certain communicative purposes.

4.4.1. Humour

Humour, according to linguists and anthropologists, is an all-encompassing categorization of things or situations which elicit amusement thus considered to be funny (Attardo, 1994: 4). For an attempt at humour to be successful, context, mood and shared background knowledge play a vital role; and although humorous expressions may be funny, they may or may not result in laughter. Paulos (1982: 24) adds on to say that the necessarily ingredient of humour is incongruity: for an assertion to be humorous it is essential that two incongruous interpretations or unusual ways of looking at something be juxtaposed or imagined simultaneously.

Humour makes use of ambiguous terms in order to generate the incongruity, and whose realization causes amusement. The semantic domains of propositions which elicit humour often necessitate the presence of ambiguity and consequently, such propositions either violate or flout Grice's maxims, particularly the maxim of quality and of manner. Dynel (2018: 86) rightly observes that the occurrence of humour may be present in propositions that observe all maxims and as a result may be regarded as contributing towards the truth of what is communicated.

Formal logic in the middle ages, according to Ziolkowski (1993: 5), was "a purely verbal art" employing natural language; hence logic and humour were never thought of as nemesis. He further argues that logic, in the most extreme communicative acts such as sophistry, can be indubitably humorous (Ziolkowski, 1993: 5). It was only until the advent of the neoteric logicians claimed that certain aspects of language, humour in this case, were flaws and therefore, sought to redesign language thus arguing that logic and humour were mutually exclusive. Logicians then stripped language off its essential aspects since the focus, in an attempt to perfect language, was placed more on the 'form,' which as a result, undermines the purpose of language by limiting it at its communicative function.

The c*haracteristica unviversalis* and the *calculus ratiocinator* are a byproduct of Leibniz's obsession with precision and exactitude which portray an oversimplified understanding of language and its functions. For him, the purpose of language is to communicate thoughts symbolically in a manner resembling mathematical notation, and to use operations of calculus, in order to assess their truth-value. This trivial replication of natural language would soon be called a universal language. Humour, as Bergen and Binsted, (2004: 79) explain, is one of the functions of a universal language beyond merely communicating thoughts, or passing judgment.

In addition, Attardo (2009: 19) says that generally speaking, humour is a universal element of language which, for the most part, is natural and yet, can also be acquired. The universality of humour is not to be understood to imply that a given specific humorous expression can be appreciated and enjoyed by all humans; but rather, that all human beings share a sense an appreciation of humorous occurrences which are not bound by the cultural or linguistic aspect of a given society since different kinds of humour (wit, satire, pun, etc.) can be appealing to different individuals.

One may find it unreasonable therefore, that humour, as a universally bound aspect of communication finds no representation in Leibniz's universal language. Lockyer and Pickering (2019: 2) shed some light on this matter by noting that the assumption has always been that humour is mere playfulness which if taken seriously, would corrupt the seriousness of serious endeavours such as mathematics.

Truth, for Galasiński (2000: 23) is an irrelevant concept in instances of humorous expressions for they are devoid of any substantial informational content, that is, they do not contribute anything to an ongoing exchange of ideas. In addition, most scholars opine that humour, in the sense of joke telling, is a result of opting out or violation of both Grice's maxims of quantity and quality (Fallis, 2009; Raskin 1985; Sweetser, 1987; Wilson and Sperber, 2000). The former maxim states that a speaker must be informative and thus refrain from saying that which contributes nothing to a conversation; and the latter requires speakers to communicate only that which is true. In light of this then, the conclusion has been that humour is useful only for the purposes of amusement and as a result, one is forced to suspend judgment whenever a joke is made since humorous propositions cannot be valued as either being true or false.

Against the view that humour is non-serious talk, Dynel (2018: 87) states that humour, despite its heterogeneous nature, often facilitates the communication of meaning which can either be true or covertly untruthful through implicates; and as such, it is erroneous therefore to divorce humorous propositions from the possibility of entailing informational content or their openness to judgment. Humorous expressions, particularly jokes, when articulated in appropriate circumstances, contribute something towards an ongoing conversation.

Ziolkowski (1993: 6) writes that fallacies and sophisma constitute logical argumentation and can be traced back to the origins of logic. He further notes that sophisma in logic can be humorous in light of the ridiculous conclusions which can be derived. Instances of a sophisma put forth by de Rijk (1967: 328) and Abelard (1970) (as cited in Stump, 2020: 105) respectively are as follows:

1. (P1) Every ass is an animal;

   (P2) But every man is an animal;

   ∴ Therefore, man is an ass

2. (P1) Every man is a stone

   (P2) Every stone is a donkey

   ∴ Therefore, every man is a donkey

The above syllogisms exemplify how humour enters logic or more specifically, how logicians can make use of logic for the purpose of being humorous. In connection with the first example, the argument is premised on knowledge about reality and yet results in an alternative conclusion different from what one would expect because of the ambiguous term ‘animal’ which can be taken to mean both ‘brute’ and ‘human.’ It is the least expected ridiculous conclusions that cause amusement, thus making the entire argument humorous.

With regard to the second syllogism, although the argument may be considered valid within a logical framework, in reality it is utter gibberish. And yet it is the very same fact of it being unintelligible that makes it humorous. Ziolkowski (1993: 12) adds on to say, “There is room for considerable humour when formal logic crashes into practical common sense— when the pretentiousness of school learning encounters the blunt thinking of home wit.” In both instances, logic is used to elicit humour, but the likes of Leibniz would not hesitate to dismiss the occurrence of humour in logic.

Humour, like ambiguity and vagueness, is regarded as an imperfection of language hence Leibniz’s universal language defaults to be exclusive of it. Leibniz’s mathematical system of logic which adopts arithmetic symbols as the language of thought on which a reasoning calculus would be operational, devalues the worth of these aspects of language. For the most part, humour is often context-sensitive and as a result, it becomes filtered out of Leibniz’s context-free formal language. In rejection of attempts to mechanize thought and arithmetize language, Ziman (1996: 6) says “But human cognition and communication are not restricted to pointer readings and algebraic formulae.” Paulos (1982: 8) concurs by saying that humour, in as much as it makes use of numerous formal devices, it cannot be reduced to mathematical equations and formulas.

From this, it follows that even if Leibniz were to acknowledge the importance of humour in language, the challenge would still remain: the representation of humour using symbols. It would be near impossible to represent jokes or any other form of humour using his binary system, [1's and 0's], or the universal characters (*a, b,* ∞,), given these are variables devoid of meaning and as such can denote anything, thus making it difficult to recognize the incongruity of humorous expressions.

Paulos (1982: 27) explains that an appreciation of humour requires one to ascend to a meta-level state, decode the implicit meanings contained in an expression, juxtapose all shades of meaning, and then contextualize the proposition. He adds on to say that these operations are much more complex and "are beyond the capabilities of computers and people who want to be computers." In light of this, it is quite evident that a mathematical system of language is limited only to explicit communicative acts of language.

Leibniz's c*haracteristica unviversalis* serves as a language whose function is merely to provide a symbolic representation of thoughts and determine their truth-value through mathematical notation. But this view of language is rather trifling— language does not function only as a means of communicating thoughts but also, as means of integrating information, building structures of meaning and navigating semantic dimensions of expressions.

4.4.2. Irony and Sarcasm

Irony is a generic term incorporating rhetorical techniques where language users conceal the intended meaning by uttering statements contrary to what is implied. An ironical proposition conveys the meaning contrary to what the proposition, when taken literally means (Searle, 1979: 115). The speaker utters proposition '*x*' and yet he or she means 'not *x*,' and like humour, irony involves incongruity. An ironic statement consists of two interpretations, the literal meaning and the metaphorical meaning, such that, upon hearing the proposition, the hearer observes that the meaning of the proposition, when taken at face value, is inappropriate to the context, and therefore infers the speaker meaning, which is the opposite of what has been said.

The presence of irony is often associated with non-observances of Grice's maxim of quality. Grice (1975: 53) argues that in an ironic utterance, the act of saying something contrary to what is actually implied results in flouting the maxim of quality. Ironic propositions resemble more of a code whose meaning can only be accurately deciphered in light of the context in which they occur rather than in isolation. For instance, in snowy weather, the proposition ‘The weather is great,’ when taken literally, it would be the case that the speaker is enjoying the weather; but when said by someone who is homeless, the intended meaning would most definitely be contradictory to what has been overtly said.

Basically, irony, like most figurative language devices such as sarcasm, hyperbole and euphemisms, can be used to exploit or flout Grice’s maxims by not overtly saying exactly what is meant. The meaning of ironic statements is mainly dependent on context and the receiver’s ability to decode the intended meaning. It would be difficult, if not impossible to resolve the irony, let alone to identify a proposition whose meaning is ironic without knowledge of the context in which it was articulated.

When human communication contains ironic expressions, misinterpretations are most likely to occur (Fariasa and Rosso2017: 114). If non-literal meanings cannot be properly decoded, inevitably, information gets misunderstood. However, misunderstandings are the price paid for effective human communication since human beings appeal to the use of non-literal forms of expression so as to express their thoughts in a rather creative and sometimes humorous manner. The incongruity between the setting or context of an utterance, and what is said and what is implied can induce humour.

Although computational linguists view irony and sarcasm as synonymous concepts (Fariasa and Rosso, 2017: 114), it is imperative to distinguish the two concepts. Sarcasm is a subtype of irony which bears a negative connotation thus expressing certain degrees of hostility directed towards the hearer. Sarcasm is defined as “cutting, contemptuous, and biting remarks, delivered often in a hostile manner” (Berger, 1993: 49). For example, the proposition ‘You are so smart,’ when taken literally, can be understood as a praise of someone’s outstanding intellectual capabilities. But if said in a context where someone has done or said something dumb, it is to be understood as a covert way of saying the said individual lacks intelligence.

A proposition expressing a sarcastic comment, when judged or understood literally, conveys a positive meaning, but the metaphorical interpretation of such a proposition carries a negative implication. The detection of a sarcastic comment is always made possible by context since, upon hearing sarcasm, the literal meaning will prove itself to be inappropriate in a given setting and as such, the hearer then realizes the real meaning and intention of the utterance which is embedded in the interpretation contrary to the actual proposition.

Generally speaking, Fariasa and Rosso (2017: 114) note that sarcasm is subsumed within irony but ironic propositions often convey a non-critical message while sarcasm tends to be critical thus convey meaning that is more derogatory and aggressive. Both irony and sarcasm contain an aspect of expectation versus reality. For both instances of irony and sarcasm, context precedes the proposition thus making it easier for the hearer to determine whether such a proposition contains either one of the two figurative devices (Woodland and Voyer, 2011: 235).

Like vagueness and ambiguity, humour, irony and sarcasm are some of the peculiarities of language which if not accurately decoded will inevitably result in misunderstandings. According to Fariasa and Rosso (2017: 113), incorporating figurative use of language in communication is a way of exploiting the interaction or connection between language and cognition which then makes it possible for people to express their thoughts in different ways so as to achieve certain communication goals. However, the indeterminacy and non-literal use of language is a forbidden phenomenon in logicism. Ironic propositions, regardless of the informational content they possess, their characteristic reversal of meaning cannot be transcribed symbolically.

Where it pertains determining the truth or falsity of ironic expressions, van Genabith (2001: 43) indicates that when judged based on their literal meaning, most, if not all, metaphorical propositions are simple false. Burke (1941: 438) illustrates this by saying that the overall formula for irony can be laid down as “what goes forth as A returns as non-A.” The apparent contrariety of meaning makes the logician cringe. Consequently, the logician, in the Leibnizian sense, opines that semantic indefiniteness characterizes the fallibility of natural language, hence a need for a universal language whose essence would ultimately be perfection.

4.5 Models of Ambiguity Resolution

Natural language speakers, as it has been explicated above, often opt to use language implicitly and as such create ambiguities. For correct and adequate comprehension of what the speaker intended to communicate, the receivers of the information ought to disambiguate any given ambiguity. It has been noted in earlier chapters that Leibniz believed that his ideal universal language is a remedy or medium through which unambiguous use of language can be realized. It is bemusing that despite the effort, ambiguity continues to prevail. It becomes more reasonable, therefore, to talk about resolving ambiguity instead of eliminating it.

Research on resolution of ambiguity in natural language is centred mainly on the processes involved in disambiguating sentences. Lin and Chen (2015: 1) indicate that studies on lexical ambiguity processing have concluded that the manner in which language users' process unambiguous and ambiguous lexical-semantic mappings differs. This is on account of the fact that when a lexical item is used ambiguously in a given proposition, for one to discern the correct reading of the entire proposition, the meaning of any such word is not to be judged in isolation but in relation with words contained in an utterance, which requires accessing different senses of meaning of an ambiguous term and also discarding inappropriate senses for the purpose of comprehension.

In order to comprehend ambiguous propositions, according to Mayberry and Miikkulainen (1994: 601), interlocutors appear "to employ automatic and immediate lexical disambiguation mechanisms even when they are compelled to alternate between two or more senses of an ambiguous word." For this reason, linguistic, psycholinguist, and computational linguistic approaches studied the processes involved in determining which semantic mapping is apt as an interpretation of a given ambiguous proposition as opposed to other competing senses of meaning.

From a linguistic point of view, there is a consensus among researchers of ambiguity resolution on the view that context plays an important role in priming the process of disambiguation (Onifer and Swinney 1981; Seidenberg et al. 1982; Conrad 1974; Hogaboam and Perfetti 1975; Simpson 1981; Tabossi et al. 1987; Ahrens 2001; Kılıçkaya 2007; Rabagliati et al. 2012; Lin and Chen 2015; Arabmofrad et al. 2022). According to these studies, context influences the process of resolving ambiguity, although at varying degrees. To illustrate the role of context in ambiguity resolution, let us take for example:

> Rumour had it that, for years, the government building had been plagued with problems. The man was not surprised when he found several spiders, roaches, and other bugs in the corner of the room (Swinney, 1979: 650).

In this instance, we recognize that the word 'bug' is ambiguous for it can either be taken to mean a generic term for insects, a hidden microphone, or a defect within a software. But when the receiver of the text processes it, he or she will be more inclined to interpret 'bugs' to denote insects.

The context of the text licenses the correct interpretation; and Mayberry and Miikkulainen (1994: 601), explain that the processing of ambiguity, when contextualized, occurs way beneath the threshold of consciousness so much so that an individual is not even aware that there was conflict of meaning.

Despite the differing views on disambiguation, a common view is that there are context-inappropriate meanings which ought to either be suppressed or discarded entirely whilst the most apt meaning is retained (Bubka and Gorfein, 1989: 4). Although context is widely accepted as important in ambiguity resolution, there is an ongoing debate on whether context has an *a priori*, a concurrent, or an *a posteriori* role in determining the correct meaning of an ambiguous proposition. The question of *when* exactly does sentential context determine the appropriate interpretation has resulted in different perspectives on how human beings process, disambiguate and comprehend language.

A number of propositions have been brought forth trying to explain how the process of disambiguation takes place; but generally speaking, there are three main models of resolving ambiguity: Context-dependent or selective access model, Exhaustive or multiple access model, and Ordered access model. These models differ in their conclusions on how meanings of ambiguous words are accessed from the human lexicon and on whether correct meaning selection occurs pre-access or post-access.

4.5.1. Context-Dependent or Selective Access Model

The context-dependent model declares that the context in which a proposition occurs determines the appropriate meaning of an ambiguous word contained in a proposition, thus resolving the ambiguity pre-access. This model hypothesizes that given an ambiguous proposition, only a single contextually befitting meaning will be accessed, and this is backed up by the assumption that context biases the selection of the most pertinent meaning (Schvaneveldt et al., 1976; Tabossi et al. 1987; Simpson, 1994).

Proponents of the selective access model argue that context precedes disambiguation in the sense that amongst the multiple meanings of an ambiguous word, a contextually congruent meaning will be activated and sustained while other inappropriate meanings remain suppressed throughout the whole process of sentence comprehension. The sentential context facilitates the lexical access of the corresponding meaning. And in any case, where it is not possible to determine the correct contextual setting because propositions are unbiased, as opposed to strongly-biased propositions, Simpson (1994: 367) argues that the primary or frequent-sensitive meaning will be accessed. An example by Mayberry and Miikkulainen (1994: 601) is as follows:

"John put the pot in the dishwasher...."

The ambiguity in this sentence results from the use of the word 'pot' which can either mean a cooking utensil or cannabis. The process of resolving the ambiguity is facilitated by the term 'dishwasher' which provides context for the statement thus making it imperative for one to interpret 'pot' in relation to a kitchen setting thus activating 'cooking utensil' as the most suitable meaning in this context. Kılıçkaya (2007: 8) notes that the effect of the context in which a proposition occurs demonstrates the interaction between accessing meaning and the processes involved in the interpretation of ambiguous sentences.

In connection to this, Orgain (2002: 2) describes the context-dependent model as an interactive model which employs an interactive mechanism between sentence context and lexical access. In light of this, the meaning of an indefinite term can instantly be selectively accessed when influenced by the context; that is to say, context comes prior to interpretation thus providing a strong bias, and by so doing, it appropriates meaning. In this way, ambiguity is resolved. The process of ambiguity resolution, according to the context model, depends entirely on the context.

Mason and Just (2007: 117) submit that ambiguity, although contained mostly in words used as such, does not warrant that an examination of language comprehension be carried out on words themselves; but that comprehension of the semantics of a proposition as a whole depends solely on sentential context. In the same vein, context, for Kılıçkaya (2007: 9), provides additional activation for accurately determining the semantics of ordinary language propositions through what is commonly referred to as the 'activation-sensitive process.'

In general terms, the context-dependent model theorists argue that ambiguity resolution is mainly a matter of selecting an appropriate meaning given the context in which a proposition occurs in. Schvaneveldt et al. (1976: 243) concur by noting that contextual cues and other words contained in an ambiguous proposition prime the applicable meaning. Let us consider another example: 'She has taken *loans* from different *banks*.' One is more inclined to select 'financial institutions' as the only meaning compatible with the semantic context primed by the word 'loans.' Hence the process of resolving ambiguity does not occur on isolated words, but rather, words are processed within the context of words (Kılıçkaya, 2007: 5).

The process of disambiguation, according to the selective-access, restricts lexical access to a single contextually biased interpretation. And subsequently, resolution occurs instantaneously provided the preceding context, meaning of words, and adequate knowledge about the world, so much that processing and comprehension occurs even before completion of the proposition (Small et al., 1988: 74). In addition, Bubka and Gorfein (1989: 4) explain that the influence of semantic context in ambiguity resolution often results in ambiguities not being recognized at all since context pre-selects only those interpretations compatible with it, thus making it unwarranted to access any other related meanings or senses of an ambiguous word.

All things considered, selective-access theory employs discourse context or frequency-sensitive mechanisms as constraints to meaning of temporarily ambiguous terms for the purposes of resolution and comprehension (Arabmofrad et al, 2022: 18). Contextual cues, therefore, eliminate equivocation by constraining meaning to a single interpretation. Simpson (1984: 372) concludes by arguing that ambiguity does not necessarily demand resolution since the context of the discourse primes contextually appropriate meanings.

4.5.2. Exhaustive or Multiple Access Model

The multiple access model theorists, in contrast to the selective-model theorists, argue that when ambiguity is encountered, all senses or meanings of an ambiguous term, without the influence of context or frequency, are activated and accessed (Ahrens 2001; Conrad 1974; Onifer & Swinney, 1981; Lin and Chen 2015). The activation of meanings of an ambiguous word is exhaustive, and as such, contextual effects do not prime the appropriate meaning at an initial stage, but rather, post-access. When all meanings have been accessed, context selects the meaning most consistent with it, and it is only thereafter that, all inappropriate meanings are discarded.

Lin and Chen (2015: 17) explain that the process of ambiguity resolution according to the exhaustive access model involves the comprehender retrieving all possible meaning of a word even if such meanings are not consistent with the context. Although meanings of an ambiguous word are activated exhaustively, Mayberry and Miikkulainen (1994: 602), point out that they are maintained at differing degrees of access levels; that is to say, the meaning which is most likely to be consistent with the context is likely to be strongly activated whilst others are activated momentarily as they await to be discarded by context.

Contrary to the context-dependent model, where context is pre-access and also pre-selects a single interpretation which best suits the discourse, the role of context for the exhaustive access model is realized *a posteriori*. Sentential context can only determine the appropriate meaning post-access. Unlike the selective access model where context biases the selection of the most one meaning, Orgain (1999: 7) notes that according to the exhaustive model of ambiguity resolution, all meanings of an ambiguous word are activated and maintained regardless of the presence of a biasing context, for this is also the case in the absence of a strong biasing context.

Since all interpretations of an ambiguity are accessed, Bubka and Gorfein (1989: 7) indicate that all inappropriate meanings would have been suppressed by the time the resolution is processed. They further indicate that the processor's decision on which meaning is correct, provided the following contextual effect, occurs rapidly so much so that even the previously activated meanings at the initial stage may not be reflected. To exemplify the process of exhaustive access model, let us take for instance the following statement:

'The government sanctions unauthorized testing of nuclear weapons'

In the above example, the word 'sanctions' is ambiguous and in line with the exhaustive accesses model, when encountered with an ambiguity, all meanings are retrieved at an initial stage. And as such, all meanings of the ambiguous term: 'prohibit a certain cause of action' and 'approval for an action' are accessed simultaneously although without commitment to any interpretation as of yet. When contextualized and taken as a whole, an analysis of this proposition would be that there are cases of authorized testing of nuclear weapons and as such, it would then follow that from the context facilitated by the term 'unauthorized' it would be reasonable to deduce that the appropriate meaning for 'sanctions' in this case is 'prohibit'; therefore, 'The government prohibits unauthorized testing of nuclear weapons.'

Generally speaking, according to the exhaustive accesses model, context does not precede lexical access but it only facilitates the process of determining meaning which corresponds with it. Orgain (2002: 8-9) states that studies advocating for exhaustive access model have argued that the process of lexical access is an automatic process independent of any contextual biases. Whenever there is an ambiguity, associated meaning are activated exhaustively without any regard to context congruity. Following the contextual selection of an apt interpretation, all other contextually irrelevant interpretations are suppressed.

4.5.3. Ordered Access Model

According to the ordered access model theorists (Conrad 1974; Hogaboam and Perfetti 1975; Gorfein & Bubka, 1989) when an ambiguity is encountered, corresponding meanings are accessed in a sequential and terminating manner. That is to say, the most frequent meaning, whether contextually congruent or incongruent, will be retrieved first; and if it is deemed incompatible with the context, it is to be discarded so that the second most frequent meaning can be retrieved.

This process of accessing meanings in the order of their frequency of usage and terminating them if they prove to be in conflict with the context continues until a more compatible meaning of the ambiguity is activated and accessed. Orgain (2002: 14) explains that since lexical access according to the ordered access model is solely dependent on frequency, the most dominant meaning will be facilitated immediately when whenever an ambiguity is present which implies that the most frequent meaning is always at an advantage of being activated at an initial stage even if it is only for the shortest time possible.

In a similar manner as the exhaustive access model, as opposed to the context-dependent access model, the ordered access model submits that lexical access is not primed or influenced by sentential access, but rather, its role in resolving ambiguity is realized after meanings associated with an ambiguity are accessed. In rebuttal to the multiple-access model, however, the ordered access model theorists argue that words are not all accessed simultaneously but rather, they are accessed in a serial and in the process, context can determine which meaning is to be adopted (Gorfein & Bubka, 1989: 7). Sentential context only filters out meanings which are irrelevant to the context in which the discourse occurs. As an illustration, consider an example put forth by Mayberry and Miikkulainen (1994: 601) which is as follows:

> "John put the pot in the dishwasher because the police were coming over for tea."

As it has already been stated earlier, the ambiguity in this proposition is perpetuated by the lexical item 'pot.' According to the ordered access model, the meaning of pot as a cooking utensil is the most dominant meaning which will be facilitated initially and maintained as the meaning consistent with the context; but as soon as the part where ‘the police were coming’ is comprehended, the initially maintained meaning is terminated in light of the context provided by ‘police.’ The subordinate or less frequent meaning of ‘pot’ which is ‘cannabis’ will be activated following the termination of the dominant meaning. Although when understood as the whole, the less frequent meaning of pot is then discarded and the comprehender oscillates back to the dominant meaning as the most appropriate in light of the fact that the police are coming for tea such that pot, in this case would mean a ‘cooking utensil’ or more specifically a ‘teapot.’

All things considered, there is a consensus between the three models of ambiguity resolution stipulating that ambiguities in natural language are resolved by context since a single sentence can have numerous interpretation which differ with the context in which they are uttered. However, the controversy is fuelled by the lack of consensus on how lexical items are accessed and the timing of contextual effects. As Gazzaniga et al. (1998: 299) explain, one cannot know which of these methodological approaches to resolving ambiguity is applicable in which cases, “but there is growing evidence that at least lexical selection is influenced by higher-level context information.”

Leibniz’s idea of formalizing language and the inferiorization of natural language, is objected to by Charlesworth, (2016: 210) who reveals that to assume that human communication and reasoning is consistent and logically flawless is quite questionable. Ambiguities and other senses of semantic indeterminacy do pervade ordinary language, and subsequently creep into natural language communicative settings. However, these can be easily resolved thus leading to effective decoding of meaning. Although natural language is deemed imperfect, it has adequate expressive powers when compared to mathematical formulae.

Sowa (2000: 349) also points out that assuming the imperfections of natural language can be remedied by merely developing an exact and perfect language is misguided and erroneous, since the apparent inconsistency of language results from the complexity of reality itself rather that the linguistics of natural language. Kosko (1994: 20) adds on to say that these complexities frustrate logicians and scientists who think reasoning is a mechanical process of determining 1’s and 0’s hence the need to abandon natural language and communicate thoughts abstractly through mathematics and symbolic logic. The mathematical language proposed by Leibniz, has, to a great extent led to the advancement of the hard sciences, such as psychics, chemistry, biology and computer sciences, where exactitude and rigor of computation is a necessary element for objectivity.

However, to communicate the conclusion reached following rigorous computations resembling those of mathematical notation, signs and characters need to be translated back into natural language for the purposes of comprehension. Semantic indeterminacy forever remains an inescapable aspect of natural language and any such form of communication is more likely to contain ambiguous or imprecise lexical items which may, to a lesser degree, affect absolute clarity of the meaning conveyed. Ambiguity cannot be eliminated but can only be resolved.

4.6 Conclusion

From the discussion in this chapter, it can be argued that semantic indeterminacy is an essential element of language which cannot be entirely eliminated from language based on an individual's will, but can be resolved for the purposes of communication. And it has not been a drawback for human communication as pioneers of a universal language seem to opine. The fuss about ambiguity in natural language and a need for an unambiguous language could be regarded as merely an attempt to demarcate science from other fields of inquiry.

Ambiguity, and all other kinds of indefiniteness, inhere in language, especially in its semantics and to some extent in the phonetics of a language. Leibniz's arbitrary decision to ban ambiguity from language results in a subtype of language, which not only restricts human communication, but also takes for granted the role of natural language in effective communication of ideas. The algebraic symbols are abstract in the sense that they do not represent any particular object and as such, can be assigned to signify anything, and that which they are to signify, is named through natural language: that is to say that the interpretations of these signs are dependent on natural language.

Using natural language, human beings encounter situations in reality for which judgments must be made, and without having to abstractly perform calculations, they have always and still do manage. Leibniz has deemed ambiguity and indefinite nature of natural language as a flaw which can lead to erroneous reasoning. The problem with this conviction is that human can reason, either with approximations or with exactitude with ease since ambiguity is often resolved and decoded meaning decoded by appeals to one's linguistic knowledge of language and to pragmatic mechanism of disambiguation such that even before a conclusion is reached, the indefiniteness would have been resolved. And the same thing occurs when, in an attempt to express thoughts in ordinary language, an ambiguity is encountered. Semantic indefiniteness can be easily addressed within the bounds of natural language in light of the models of ambiguity resolution proposed without any need to appeal to an abstract mathematical language in the name of precision.

Chapter Five: Conclusion

This chapter presents the summary of the entire research by highlighting the main arguments raised in each chapter, in light of which the cardinal conclusion follows. The study sought to investigate the capabilities of Leibniz's universal language in eliminating ambiguities, encapsulating all other aspects of natural language, and mirroring reality in a definite manner.

Chapter one of this study provided the background of the concerns raised by Leibniz's attempt to mechanize thought and mathematize language. Chapter two outlined Leibniz key arguments where it pertains natural language and its imperfections which led to his idea of developing an ideal universal language and a reasoning calculus. The objective of this chapter was to attain an understanding of Leibniz's project and assess it feasibility in practice. The chapter concluded that Leibniz's analysis of natural language which appreciates semantic indeterminacy as an inherent feature of natural language was accurate.

Chapter three demonstrates the capabilities and application of an unambiguous form of communication embodied within Leibniz's *characteristica universalis* and the level of certainty guaranteed by the computations in light of the *calculus ratiocinator*. Leibniz's project has found its realization mainly in automated reasoning where theorems can be expressed in formal languages; and following which, automated mathematical computations and manipulations can be performed to draw conclusions. This chapter also acknowledges the importance of mathematizing natural language in areas of study concerned with the hard sciences where pernickety, rigorous communication of ideas, and accurate computations are a necessary element for reaching a consensus about statements of fact about reality.

This chapter concludes that Leibniz's universal language and its complimentary reasoning calculus, have been, to a great extent, successful in suppressing semantic indeterminacy, and have played a major role in the construction of algorithms and computing programs for automated *reasoners*. Nevertheless, the idea of mathematizing natural language in an attempt to do away with ambiguity and vagueness, finds its deficiency in natural discourse settings where human communication remains highly depended on natural languages. Although Leibniz had hoped to eliminate ambiguity for the purposes of precision, ambiguity persists being pervasive. Ambiguity then is understood to be an inescapable, although resolvable, attribute of natural language.

The study recognizes that ambiguity is an inherent characteristic of natural language. However, it is not a static feature which could serve as sufficient reason for language users to be compelled to step outside of ordinary language for the sake of exactitude. Chapter four presents an understanding of semantic indeterminacy and how natural language users are capable of comprehending the meaning conveyed despite the presence of ambiguity or vagueness of meaning. This chapter argues that natural language is not in-itself ambiguous, but rather, the ambiguity mostly arises when language users deliberately use language ambiguously in order to achieve desired ends.

Certain pragmatic elements such as the background knowledge shared by interlocutors, the environmental setting in which the discourse occurs, and the relation which speakers have, play a role in eliminating any misunderstandings or inaccuracies that may occur in the process of communication. Observations within this chapter indicated that ambiguity is not an irresolvable complication of natural language, but that, it is a peculiar characteristic of natural language resolvable through contextualization.

The key focus in chapter four was also to bring to light the linguistic models of ambiguity resolution, which are often taken for granted or obscured from those who possess inadequate knowledge about reality and the language which reflects it. Just as signs and symbols of Leibniz's universal language and a reasoning calculus are thought to make a symbolic language more precise, the pragmatic context specifies meaning in natural language. Interlocutors make use of literary devices in natural language in order for them to articulate and express meaningful thoughts about reality in way that the intended meaning is not explicitly conveyed, and this is something that symbols cannot adequately capture.

In as much as Leibniz's mathematical language cannot entirely capture or express natural language meanings conveyed through non-literal use of language, it has played a significant role in the advancement of technology and scientific knowledge. The precision of a mathematical language and the degree of certainty guaranteed by mathematical computations are two aspects which make Leibniz's program of eliminating ambiguity from language attractive since thoughts can be expressed without fear of misinterpretation or misunderstandings given that all signs and symbols have been ascribed meanings unique to each sign. However, in a natural setting, communication between human beings is not entirely symbolic and precise in a Leibnizian sense.

And although non-verbal communication plays a role in deducing the intended meanings of propositions, such propositions are articulated using natural language because a mathematized language cannot encapsulate all aspects of natural language in a manner which allows it to be a substitute for natural language in ordinary discourse. Chapter four concluded that Language is not simply a tool of for communicating straightforward expressions but serves as an instrument for fulfilling certain other functions common amongst human beings: some of which cannot be symbolized arithmetically.

The primary conclusion of this study is that the importance of a symbolic language and a reasoning calculus can only be realized within the hard sciences where conjectures and theorems are subjected to rigid computations in order to establish proofs establishing their truth or refutations exposing faults. For any such computations to commence, a mathematical language proves necessary for mathematical notation procedures to be carried out. Arithmetization of natural language has contributed immensely in areas of study such as robotics, artificial intelligence, and physics, to name a few. This highlights the power of precision and exactitude in expressing ideas and computing epitomized by a mathematical language.

Notwithstanding the capabilities of a logico-mathematical language, the development of algorithmic programs for automatic theory provers and automatic reasoners, in practice, cannot serve as a language for communicating thoughts amongst humans. It serves as a perfect language for human-machine communication, and on the ground that it fails to accommodate certain aspects of ordinary language which contribute towards effective communication thoughts, it is inadequate for adequately transmitting both implicit and explicit meanings between human interlocutors.

The arguments raised in this study do are not aimed at denouncing Leibniz's mathematical language but seek to bring to light its shortcomings in eliminating ambiguities in natural language and in serving as the principal means of human communication. From this, it follows that instead of fabricating technical languages with hopes of eradicating ambiguity, semantic indeterminacy, in general, should be acknowledged as a distinct feature of natural language. When users of natural language use language implicitly, the intended meaning can always be deduced.

In light of this, ambiguity is not as much of a defect as Leibniz had portrayed it to be, but rather, a characteristic feature of language which, often at times, is resolved even without interlocutors ever being aware that there was conflict of meaning in their utterances. Different meanings of a single lexical item result from a word being used in different domains, and yet, in each specific application, meaning is exact, hence ambiguity is seldom a problem for natural language speakers. The problem, therefore, is not necessarily with natural language but with those who think that human reasoning cannot be flawed hence their need to not only mechanize thought but also language.

References

Aberth, O. (2001). *Computable Calculus.* Academic Press.

Ahrens, K. (2001). On-line Sentence Comprehension of Ambiguous Verbs in Mandarin. . *Journal of East Asian Linguistics*, 337–358.

Al Kashari, Z. & Al Taheri, F. (2019). Automated Reasoning and Robotics . *American Scientific Research Journal for Engineering, Technology, and Sciences (ASRJETS),* 91-100.

Anellis, H. I. (2015). *Peirce's Role in the History of Logic: Lingua Universalis and Calculus Ratiocinator. In The Road to Universal Logic. Eds. Koslow, A. & Buchsbaum, A.* Springer International Publishing Switzerland.

Annoni, M. (2006). Implications of Synechism: Continuity and Second-Order Vagueness. *Cognitio-Estudos. Vol. 1, No.3*, 96-108.

Antognazza, M. R. (2018). *The Oxford Handbook of Leibniz.* United States of America: Oxford University Press .

Antognazza, M. R. (2016). *Leibniz: A Very Short Introduction.* Oxford University Press, 2016.

Arabmofrad, A. B. (2022). Transformational Ambiguity Resolution in Persian. *Journal of Critical Applied Linguistics Studies* , 16-31.

Attardo, S. (1994). *Linguistic Theories of Humor* . Berlin: New York: Mouton de Gruyter .

Attardo, S. (2009). *The Routledge Handbook of Language and Humor.* London: New York: Routledge: Taylor & Francis Group.

Austin, J. L. (1962). *Sense and Sensibilia, Warnock, G.J. (ed.).* Oxford: Oxford University Press.

Bergen, B. & Binsted, K., (2004). *The cognitive linguistics of scalar humor. In M. Achard & S. Kemmer (Eds.), Language, Culture, and Mind.* Stanford, CA: CSLI.

Berger, A. A. (1993). *An anatomy of humor.* Transaction Publishers.

Boole, G. ((1916[1951])). *The Laws of Thought.* Open Court Publishing Company.

Boole, G. (1848). The Calculus of Logic. *Cambridge and Dublin Mathematical Journal*, 183–198.

Boole, G. (1854). *An Inverstigation of the Laws of Thought.* London, Cambridge: McMillan and Co.

Bronowski, J. (2008). *The Origins of Knowledge and Imagination.* Yale University Press.

Burke, K. (1941). Four Master Tropes. *The Kenyon Review, Vol. 3, No. 4* , 421-438.

Burns, L. C. (1991). *Vagueness: An Investigation into Natural Languages and the Sorites Paradox.* Kluwer Academic Publishers. .

Byers, W. (2011). *The Blind Spot: Science and the Crisis of Uncertainty.* Princeton, New Jersey: Princeton University Press.

Carnap, R. (1937). *Logical Syntax of Language.* London: Routledge.

Casanave, A. L. (2012). *Symbolic Knowledge From Leibniz to Husserl.* London: UK: Collage Publications.

Charlesworth, A. (2016). A Theorem about Computationalism and ‘‘Absolute’’ Truth. *Minds & Machines*, 205–226.

Chierchia, G. &. McConnell, G. (1990). *Meaning and Grammar.* London, England: The MIT Press.

Conrad, C. (1974). Context effects in sentence comprehension: A study of the subjective lexicon. *Memory and Cognition*, 130–138.

Couturat, L. (2006). *The Algebra of Logic.* eBook: www.gutenberg.net.

Dascal, M. (1987). *Leibniz. Language, Signs and Thought.* Philadelphia: John Benjamins Publishing Company.

Davies, B. (2000). Grice’s Cooperative Principle: Getting the Meaning Across. *Working Papers in Linguistics*, 1-26.

Deemter, K. (2010). *Not Exactly: In Praise of Vagueness.* USA: New York: Oxford University Press Inc.

Descartes, R. (1981). *Philosophical Letters. (Ed.) Kenny, A.* Clarendon Press.

Dick, R. (1995). *Legal Drafting in Plain Language.* Carswell.

Dynel, M. (2018). *Irony, Deception and Humour: Seeking the Truth about Overt and Covert Untruthfulness.* Boston/Berlin: Walter de Gruyter, Inc.

Eco, U. (1995). *The Search for the Perfect Language.* German: Munich: Blackwell Publishers Ltd.

Fariasa, I. H. (2017). *Irony, Sarcasm, and Sentiment Analysis.* Elsevier Inc.: 113-128.

Frege. (1879 [1997]),). *Begriffsschrift.* Translated by T. W. Bynum.

Feyerabend, P. (1993). *Against Method.* Verso.

Fine, K. (1975). Vagueness, Truth and Logic. *Synthese*, Vol. 30, No. 3/4, 265-300.

Frazier, O. (2002). Context Effects and Lexical Ambiguity Processing: An Activation-Based Account. *Master's Theses. Paper 810.*

Frege, G. (1982). *Uber den Zweck der Begriffsschrift.* Oxford: Blackwell.

Galasiński, D. (2000). *The Language of Deception.* Thousand Oaks: Sage.

Gardent, C. & Webber, B. (2001). Towards the Use of Automated Reasoning in Discourse Disambiguation. *Journal of Logic, Language, and Information. Vol. 10, No. 4*, 487-509.

Garzon, M. (1989). Neural Computability II:. *IJCNN International Joint Conference on Neural Networks*, 631-637.

Gazzaniga, M. I. (1998). *Cognitive Neuroscience: the Biology of The Mind.* New York : Norton.

Godil, S. S. (2011). neurosciences? *Surgical Neurology International*, 1-10.

Goldstein, R. (2005). *Incompleteness: The Proof and Paradox of Kurt Godel.* New York: London: W. W. Norton & Company, Inc.

Gorfein, D. (1989). *Resolving Semantic Ambiguity.* New York: Springer-Verlag New York Inc.

Hadi, A. (2013). A Critical Appraisal of Grice's Cooperative Principle. *Open Journal of Modern Linguistics. Vol.3, No.1.*, 69-72 .

Hang, H. (1960). Toward Mechanical Mathematics. In J. &. Siekmann, *Automation of Reasoning: Classical Papers on Computational Logic 1957-1966* (pp. 244-264). New York: Springer-Verlag Berlin Heidelber.

Harel, D. (1992). *Algorithmics: The Spirit of Computing. Ed. Roni Rosner, R.* Addison-Wesley.

Harrison, j. (2009). *Handbook of Practical Logic and Automated Reasoning.* Cambridge: New York: Cambridge University Press.

Heijenoort, J. (1967). Logic as Calculus and Logic and Language. *Synthese*, 324-330.

Hempel, C. G. (1939). Vagueness and Logic. *Philosophy of Science, Vol. 6, No. 2.*, 163-180.

Herder, J. G. (1799). *Verstand und Erfahrung: Eine Metakritik \ur Kritik der reinen Vernunft. Vol. 2.* The Bavarian State Library.

Hintikka, J. (1997). *Lingua Universalis vs. Calculus Ratiocinator.* Boston: Kluwer Academic Publishers.

Hogaboam, T. &. (1975). Lexical Ambiguity and Sentence Comprehension. . *Journal of Verbal Learning and Verbal Behavior*, 265–274.

Jacquette, D. (2019). *Frege: A Philosophical Biography.* United Kingdom: Cambridge University Press.

Jolley, N. (1995). *The Cambridge Companion to Leibniz.* Cambridge University Press.

Kanterian, E. (2012). *Frege: A Guide for the Perplexed.* London: New York: Bloomsbury Publishing Plc.

Katz, J. & Fodor, J. (1963). The Structure of a Semantic Theory. *Language*, 170-210

Keith, A. (1986). *Linguistic Meaning.* London: Routledge.

Kılıçkaya, F. (2007). Lexical Ambiguity Resolution: Word processing, Recognition and Context Effect. 1-12.

Kleene, S. C. (1980). *Introduction to Metamathematics.* Oxford: Walters-Norddoff Publishing.

Konopka, K. (2007). *Systems Biology: Principles, Methods, and Concepts.* London: New York: Taylor & Francis Group, LLC.

Korner, S. (1962). *The Philosophy of Mathematics: An Introduction.* New York: Harper & Brothers.

Kopytowska, M. & Chilton, P. (2018). *Religion, Language, and the Human Mind.* Oxford University Press.

Kosko, B. (1994). *Fuzzy Thinking: The New Science of Fuzzy Logic.* Great Britain: HarpcrCollins Publishers.

Kreeft, P. (2010). *Socratic Logic: A Logic Text using Socratic Method, Platonic Questions & Aristotelian Principles. Edited by Trent Dougherty*. USA: St. Augustine's Press.

Kristensen, T. (2021). *Artificial Intelligence: Models, Algorithms and Applications.* UAE: Bentham Science Publishers.

Kusch, M. (1989). *Language as Calculus vs. Language as Universal Medium: A Study in Husserl, Heidegger and Gadamer.* The Netherlands: Kluwer Academic Publisher.

Lacona, A. (2018). *Logical Form-Between Logic and Natural Language.* Italy: Turin: Springer International Publishing AG.

Ladegaard, H. J. (2009). Pragmatic Cooperation Revisited: Resistance and Non-cooperation as a Discursive Strategy in Asymmetrical Discourses. *Journal of Pragmatics*, 649–666.

Lakatos, I. (1977). *Proofs and Refutations: The Logic of Mathematical Discovery.* London: New York: Cambridge University Press.

Leibniz, G. (1989). *Philosophical Essays. Ed. and rans. by Ariew, R. & Garber, D.* Cambridge: Hackett Publishing Company.

Leibniz, G. W. (1956). *Philosophical Papers and Letters.* USA: The University of Chicago Press.

Leibniz, G. W. (1989). *Philosophical Papers and Letters: A Selection.* Netherlands: Springer Science & Business Media.

Leibniz, W. G. (1996). *New Essays on Human Understanding. Trans. Remnant, P. & Bennett, J.* New York: Cambridge University Press.

Lichtenberg, C. G. (1800-06). *Sudelbücher.* Trans: Zohn, H. : Insel Verlag.

Lin, C. & Chen, Y. (2015). Exhaustive Semantic Activation for Reading Ambiguous Verbs in Chinese Sentences. *Lingua Sinica. Vol. 1, No. 17*, 1-23.

Levinson, S. C. (1983). Pragmatics. Cambridge: Cambridge University Press.

Liuxiang, H. (2000). *Leibniz's Ideal of Characteristica Universalis. in Das Neueste über China: G.W. Leibnizens Novissima Sinica von 1697 : Internationales Symposium, Berlin 4. bis 7. Oktober 1997.* Berlin: Franz Steiner Verlag.

Lobeck, A. & Denham, K. (2006). *Language in the Schools: Integrating Linguistic Knowledge Into K-12 Teaching.* Routledge.

Lockyer, S. & Pickering, M. (2019). You Must Be Joking: The Sociological Critique of Humour and Comic Media. *Sociology Compass. Vol. 2, No. 3*, 1-10.

Mason, R. & Just, M. (2007). Lexical Ambiguity in Sentence Comprehension. *Brain Research*, 115-127.

Mayberry, R. M. (1994). Lexical Disambiguation Based on Distributed Representations of Context Frequency. In A. &. Ram, *Proceedings of the Sixteenth Annual Conference of the Cognitive Science Society* (pp. 601-606). New York: Routledge.

Medina, J. (2005). *Language: Key Concepts in Philosophy.* London: New York: continuum.

Millan, J. B. (2021). The Leibnizian Background of the Frege-Schroder Polemic. *The Review of Symbolic Logic*, 411-446.

Ndungi, R. & 'Uyun, S. (2023). A Review of Automated Reasoning And Its Applications in The 21st Century. *Indonesian Journal of Computer Science*, 483-492.

Nykiel, A. R. (2007). *Handbook of Marketing Research Methodologies for Hospitality and Tourism.* Psychology Press.

Onifer, W. & Swinney, D. (1981). Accessing lexical Ambiguities during Sentence Comprehension: Effects of frequency of meaning and contextual bias. *Memory and Cognition*, 225–236.

Paleo, B. W. (2016). Leibniz's Characteristica Universalis and Calculus Ratiocinator Today. In C. Tandy, *300 Years After Leibniz* (pp. 1-20).

Paleo, W. B. (2016). Leibniz's Characteristica Universalis and Calculus Ratiocinator Today.

Paulos, J. A. (1982). *Mathematics and Humor.* United States of America : University of Chicago Press.

Peckhaus, V. (2004). Calculus Ratiocinator vs. Characteristica Universalis? The Two Traditions in Logic, Revisited. *History and Philosphy of Logic*, 1-15.

Peirce, C. (2011). *Philosophical Writings of Peirce. Ed. Buchler, J.* New York: Dover Publications, Inc.

Pinkal, M. (1995). *Logic and Lexicon: The Semantics of the Indefinite* . Kluwer Academic Publishers .

Pombo, O. (1987). *Leibniz and the Problem of Universal Language.* Nodus Publikationen Münster.

Rabagliati, H. P. (2012). Top-Down Influence in YoungChildren's Linguistic Ambiguity Resolution. . *Developmental Psychology*, 1-16.

Raffman, D. (2014). *Unruly Words: A Study of Vague Language.* Oxford: New York: Oxford University Press.

Russell, B. (1923). Vagueness. *The Australasian Journal of Psychology and Philosophy*, 84-92.

Saeed, J. (2003). *Semantics* . London: Blackwell.

Sarkar, S. (2021). *Logical Empiricism at its Peak: Schlick, Carnap, and Neurath.* New York: Routledge .

Savas, L. T. (1994). *Foundations of Speech-Act Theory. London: Routledge.* London: Routledge.

Schmitz, H. N. (2011). *Vagueness in Communication.* London: New York: Springer Heidelberg Dordrecht.

Schwindt, J. (2022). *Universe Without Things: Physics in an Intangible Reality.* Germany: Berlin: Springer-Verlag.

Searle, J. (1979). *Expression and Meaning.* Cambridge: Cambridge University Press.

Seidenberg, S. T. (1982). Automatic Access of the Meanings of Ambiguous Words in Context: Some Limitations of Knowledge-Based Processing. *Cognitive Psychology*, 489–537.

Shostak, R. (2011). *7th International Conference on Automated Deduction: Proceedings.* New York: Springer.

Shuy, R. (2001). *Discourse analysis in the legal context: The Handbook of discourse analysis.* Blackwell Publishers.

Sidebotham, H. T. (2003). *The A to Z of Mathematics: A Basic Guide.* John Wiley & Sons.

Siekmann, J. & Wrightson, G. (1983). *Automation of Reasoning: Classical Papers on Computational Logic 1957-1966.* Germany: Springer-Verlag Berlin Heidelberg.

Simatupang, S. M. (2007). How Ambiguous is the Structural Ambiguity. *Jurnal Lingua Cultura Vol.1, No.2* , 99-104.

Simpson, G. (1981). Meaning Dominance and Semantic Context in the Processing of Lexical Ambiguity. *Journal of Verbal Learning and Verbal Behavior*, 120–136.

Simpson, G. B. (1994). Context and the Processing of Ambiguious Sentences. In M. Gernsbacher, *Handbook of Psycholinguistics* (pp. 359-374). Elsevier.

Small, S. C. (1988). *Lexical Ambiguity Resolution: Perspectives from Psycholinguistics, Neuropsychology, and Artificial Intelligence* . San Mateo: California: Morgan Kaufmann Publishers, Inc.

Sowa, F. (2000). *Knowledge Representation: Logical, Philosophical, and Computational Foundations.* United States of America: Brooks/Cole.

Strickland, L. & Lewis, H. (2022). *Leibniz on Binary: The Invention of Computer Arithmetic.* Cambridge, Massachusetts: The MIT Press.

Stump, E. (2020). *Dialectic and Its Place in the Development of Medieval Logic.* Cornell University Press.

Swinney, D. A. (1979). Lexical Access During Sentence Comprehension: (Re)consideration of Context Effects. . *Journal of Verbal Learning and Verbal Behavior*, 645-659.

Tabossi, P. C. (1987). Accessing lexical ambiguity: Effects of context and dominance. *Psychological Research*, 161–167.

Taha, A. K. (1983). Types of Syntactic Ambiguity in English. *IRAL*, Vol. XXI/4. 251–266.

Thomas, J. (1998). Cooperative principle. *In: Concise Encyclopedia of Pragmatics. Ed. Mey, J. Elsevier, Amsterdam*, 176–179.

Trauth, G. P. (1989). *Towards an analysis of Humboldt's 'inner language form', in: I. Rauch and G. F. Carr (eds.), The Semiotic Bridge: Trends from California.* Berlin, New York: Mouton de Gruyter, 409-420.

Truemper, K. (2010). *Effective Logic Computation.* Plano: Texas: Leibniz.

Tsohatzidis, S. (1994) Foundations of Speech Act Theory: Philosophical and Linguistic Perspectives. London: Routledge

Turing, A. & Copeland, J. (2004). *The Essential Turing: Seminal Writings in Computing, Logic, Philosophy, Artificial Intelligence, and Artificial Life plus. The Secrets of Enigma.* United States: New York: Oxford University Press Inc.

Turing, A. (1936). On Computable Numbers with an Application to the Entscheidungsproblem. *Proceedings of the London Mathematical Society. Vol. 2, No. 42*, 230-265.

Turing, A. (1948). *Intelligent Machinery. In The Esssential Turning, Copeland (ed.)* . Oxford University Press: National Physical Laboratory.

Ullmann, S. (1962). *Semantics: an Introduction to the Science of Meaning.* Oxford: Basil Blackwell.

van Dijk, T. (2008). Context Theory and the Foundation of Pragmatics. 1-13.

van Genabith, J. (2001). Metaphors, Logic, and Type Theory. *Lawrence Erlbaum Associates, Inc. Vol. 2, No. 16*, 43–57.

Vogt, A. & Barta, J. (1997). *The Making of Tests for Index Numbers: Mathematical Methods of Descriptive Statistics; Published in Honor of the 50th Anniversary of the Death of Irving Fisher*. Berlin: Springer-Verlag.

Voronkov, A. (2003). Automated Reasoning: Past Story and New Trends. . *International Joint Conference on Artificial Intelligence*, 1607-1612.

Weaver, A. J. (2001). Introduction: (Post) Modern Science (Education): Propositions and Alternative Paths. *Counterpoints, Vol. 137*, 1-22.

Whitehead, N. A. (1910). *Principia Mathematica.* Cambridge: Cambridge University Press.

Wierzbicka, A. (2001). *Leibnizian linguistics, In Perspectives on Semantics, Pragmatics, and Discourse. By Kenesei, I. and Harnish,R.* Philadelphia: John Benjamins Publishing Company.

Williamson, T. (1994). *Vagueness.* London: Routledge.

Winkler, S. (2015). *Ambiguity: Language and Comunication.* Berlin: De Gruyter.

Wittgenstein, L. ((1921[1999])). *Tractatus Logico-Philosophicus. Transleted by Ogden Mineola'.* New York: Dover Publications.

Wittgenstein, L. (1958). *Philosophical Investigations. Trans: G. E. M. Anscombe.* Oxford: Basil Blackwell Ltd .

Wolfgang, B. & Schmitt, P. (2013). *Automated Deduction - A Basis for Applications Volume I Foundations - Calculi and Methods Volume II Systems and Implementation Techniques Volume III Applications.* Springer Science & Business Media.

Woodland, J. & Voyer, D. (2011). Context and Intonation in the Perception of Sarcasm. *Metaphor and Symbol, Vol. 3, No.26 227-239*, 227-239.

Wos, L. & Pieper, G. (2000). *The Collected Works of Larry Wos: Exploring the Power of Automated Reasoning.* Singapore: World Scientific Publishing Co. Pte. Ltd.

Wos, L. (1987). The Problem of Choosing the Inference Rule to Employ. . *Journal of Automated Reasoning*, 201-209.

Wos, L. (1994). Logical Basis for the Automation of Reasoning: Case Studies. In M. H. Gabby, *Handbook of Logic in Artificial Intelligence and Logic Programing* (pp. 41-125). Clarendon Press.

Wurm, C. (2021). Reasoning with Ambiguity. *Journal of Logic, Language and Information* , 139–206.

Yoos, G. (2016). *Simplifying Complexity: Rhetoric and the Social Politics of Dealing with Ignorance.* . Poland: De Gruyter.

Zhang, J. (2007). *Visualization for Information Retrieval.* Berlin: Springer Berlin Heidelberg.

Ziman, J. (1996). *Reliable knowledge: An Exploration o f the Grounds for Belief in Science.* Great Britain : Cambridge University Press .

Ziolkowski, J. M. (1993). The Humour of Logic and the Logic of Humour in the Twelfth-Century Renaissance. *The Journal of Medieval Latin. Vol. 3.*, 1-26.

Cuacuh, M. (2012). More than One Meaning. *Зборник радова Филозофског факултета XLII (1)*, 231-248.